YEARS 11&12

MATHEMATICS SPECIALISED

TEACHING & LEARNING SUPPLEMENT

INDEPENDENT SCHOOLS TASMANIA

TASMANIAN CATHOLIC education office

Tasmanian Government

Teaching and Learning Supplement
MATHEMATICS SPECIALISED (MTS415118)

ADVICE TO TEACHERS

This document helps to describe the nature and sequence of teaching and learning necessary for learners to demonstrate achievement of course outcomes.

It suggests appropriate learning activities to enable learners to develop the knowledge and skills identified in the course outcome statements.

Tasks should provide a variety and the mix of tasks should reflect the fact that different types of tasks suit different knowledge and skills, and different learning styles. Tasks do not have to be lengthy to make a decision about learner demonstration of achievement of an outcome.

COURSE SPECIFIC ADVICE

This Teaching and Learning Supplement for *Mathematics Specialised* must be read in conjunction with:

- Mathematics Specialised (MTS415118) course information from the TASC website (live)
- Mathematics Specialised (MTS415118) External Assessment Information Sheet (2018)
- Mathematics Methods (MTM415117) External Assessment Information Sheet (2018)
- Use of Calculator Policy 2017 Information Sheet (2017)

The documents listed above can be found using the following links:

MTS415118 materials: https://www.tasc.tas.gov.au/students/courses/mathematics/mts415118/

MTM415117 materials: https://tasc.tas.gov.au/students/courses/mathematics/mtm415117/

This Teaching and Learning Supplement for *Mathematics Specialised* contains advice to assist teachers delivering the course and can be modified as required. It is designed to support teachers new to or returning to teaching this course.

The *Mathematics Specialised* course provides learners width a breadth of mathematical experiences that are designed to build understanding, fluency, problem solving and reasoning. In order to undertake this course it is expected that learners have successfully studied *Mathematics Methods* 4 in year 11 or *Mathematics Methods Foundation* 3 in year 11 and concurrently be studying *Mathematics Methods* 4 in year 12.

Mathematics Specialised is a pure mathematics course which aims to further develop learners'

- understanding of concepts and techniques drawn from sequences and series, matrices and linear algebra, complex numbers, differential and integral calculus
- ability to solve applied problems and undertake applications involving the concepts and techniques drawn from sequences and series, matrices and linear algebra, complex numbers, differential and integral calculus
- reasoning in mathematical contexts and interpretation of mathematical information, including ascertaining the reasonableness of solutions to problems and applications
- capacity to communicate in a concise and systematic manner using appropriate mathematical language
- capacity to choose and use technology appropriately and efficiently

SEQUENCE OF CONTENT

Mathematics Specialised is divided into four compulsory units of study

Unit 1: Sequences and Series (5 – 6 weeks)

Unit 2: Complex Numbers (5 – 6 weeks)

Unit 3: Matrices and Linear Algebra (5 – 6 weeks)

Unit 4a: Calculus – differentiation (5 – 6 weeks)

Unit 4b: Calculus – integration and further application of differentiation (5 – 6 weeks)

Course Delivery

- recommended time spent on each unit is indicated in brackets.
- units can be taught in any order, though it is recommended that the sub-units within a unit be taught in the specific order as shown in the *Mathematics Specialised* course document.
- It is recommended that the delivery of the course focus on:
    - inquiry and problem solving to develop skills and understanding
    - embed formative assessment practices in identifying understanding and to provide appropriate support for learner learning
    - group activities to build understanding
    - using real-world examples to extend learning through the applications

TEACHING AND LEARNING

Unit 1
Sequences and Series

Learners will study a range of sequences by learning about their properties, meet some of their important uses and begin to understand the ideas of convergence and divergence and develop some methods of proof:

Examples of learning activities:

- In class activities – including individual and group activities to develop conceptual understanding and skill development. Involving such questions as:

  Sum $\frac{4^2 5}{3} + \frac{5^2 7}{3} + \frac{6^2 9}{3} + \ \ldots\ldots$ to n terms

  Use mathematical induction to prove that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ for all positive integers

  Prove that the sequence $\left\{\frac{3n-2}{5}\right\}$ diverges to infinity

  Further activities can be found in *Activities for Pairs and/or Groups*

- Assignments – covering the concepts and applying that learning through short answer and extended answer questions. Using problem solving strategies to determine the answer and discuss the appropriateness of the answer

- Unit test – Developing time management skills and to solve a range of problems within the topic without guidance

- Application(s)
  - i. Koch snowflake (see Examples of Applications)
  - ii. Mathematical induction involving inequalities or recursively defined sequences and series
  - iii. Monotonicity and boundedness of sequences
  - iv. Convergent nature of $\frac{1}{n!}$

Unit 2 Complex Numbers

Learners will be introduced to a different class of numbers and understand that such numbers can be represented in several ways and that their use allows factorisation to be carried out more fully than was previously possible.

Examples of learning activities:

- In class activities – including individual and group activities to develop conceptual understanding and skill development. Involving such questions as:

  Solve $Z^5 + 1 = 0$. Present solutions in polar form.
  Hence solve $Z^9 - 16Z^5 + Z^4 - 16 = 0$

  If the principal argument of the complex number $1 + ai$ is $-\frac{\pi}{6}$, find the value of the real number a

  Sketch the region of the Argand Plane which represents complex numbers Z, where $|Z + 2 - i| > 2$

  In pairs, find as many possible functions that satisfy the following:
  $\mathrm{f}(i) = 4 + 3i$
  $\mathrm{f}(2 - i) = 7 + 2i$
  $\mathrm{f}(i) = g(1 + i)$

  Further activities can be found in *Activities for Pairs and/or Groups*

- Assignments – covering the concepts and applying that learning through short answer and extended answer questions. Using problem solving strategies to determine the answer and discuss the appropriateness of the answer

- Unit test – Developing time management skills and to solve a range of problems within the topic without guidance

- Application(s)
  i. Graphs of functions in polar form
  ii. Mandelbrot set (see Examples of Applications)
  iii. Investigation into Regular Polygons produced by Complex Numbers

Unit 3
Matrices and Linear Algebra

Learners will be introduced to new mathematical structures and appreciate some of the ways in which these structures can be used and applied. Trigonometric identities should be used to develop ideas in matrices and linear transformations.

Examples of learning activities:

- In class activities – including individual and group activities to develop conceptual understanding and skill development. Involving such questions as:

  Matrices $\mathbf{A}$ and $\mathbf{I}$ are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
  Show that $\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$

  Find the curve whose image is $\frac{x^2}{4} + \frac{y^2}{9} = 1$ when the transformation $\mathrm{L}(x, y)\colon (2x, 3y)$ is applied. Hence find the enclosed area.

  Consider the system of equation, in which a, b and c are real numbers:

  $x - y + z = a$
  $x + y + 9z = b$
  $2x - y + 6z = c$

  Show that $3a + b - 2c = 0$ *is* required for the system of equations to have a solution in x, y and z. *If* $a = 1, b = 5$ *and* $c = 4$, state the solution to the system of equations and provide a geometrical interpretation.

  In pairs, multiply the matrix by each postion vector of the triangle (see attached sheet at end Unit 3). Plot the new position of the triangle and state the transformation that the matrix represents.

  Further activities can be found in *Activities for Pairs and/or Groups*

- Assignments – covering the concepts and applying that learning through short answer and extended answer questions. Using problem solving strategies to determine the answer and discuss the appropriateness of the answer

- Unit test – Developing time management skills and to solve a range of problems within the topic without guidance

- Application(s)
  i. Markov chains or Leslie Matrices (see Examples of Applications)
  ii. Matrices and modelling in food webs
  iii. Modelling stock market probabilities
  iv. Cryptography (see Examples of Applications)

Unit 4a
Calculus - differential

Learners will extend their existing knowledge and understanding of this vital branch of mathematics by developing a greater capacity for integrating functions and by introducing simple differential equations and their uses. Learners will focus on:

- implicit differentiation and its use in finding tangents and normal to curves
- derivatives of inverse trigonometric functions, a^x and $\log_a x$ and composites of these
- applications of first and second derivatives to curve sketching
- on the Fundamental Theorem of Calculus (review)
- properties of definite integrals, including then application of definite integrals to find areas under or between curves and to find volumes of solids of revolution about either the x-axis or the y-axis

Examples of learning activities:

- In class activities – including individual and group activities to develop conceptual understanding and skill development. Involving such questions as:

  For the curve with the equation $x^3 + y^3 = 4xy$,
  Find $\dfrac{dy}{dx}$ and then find the equation of the tangent to the curve at the point $(2,2)$

  Consider the function $f(x) = (x+1)e^{x-1}$,
  (a) Show that $x = -1$ is the ony zero
  (b) Show that $\left(-2, -\dfrac{1}{e^3}\right)$ is the only stationary point, global minimum
  (c) Find and classify all points of inflection
  (d) Sketch the graph of the function $f(x) = (x+1)e^{x-1}$

  Further activities can be found in *Activities for Pairs and/or Groups*

- Assignments – covering the concepts and applying that learning through short answer and extended answer questions. Using problem solving strategies to determine the answer and discuss the appropriateness of the answer

- Unit test – Developing time management skills and to solve a range of problems within the topic without guidance

- Application(s)
  i. Debate: "Who invented calculus?"
  ii. Volumes of common 3D objects e.g. bottles, doughnuts (see Examples of Applications)
  iii. The trapezoidal Rule and its use to approximate the area under the curve of a non-integral function (see Examples of Applications)

Unit 4b
Calculus - integral

Learners will focus on:

- Techniques of integration, including:
  i. integration of functions with linear arguments
  ii. trigonometric identities
  iii. inverse trigonometric functions
  iv. partial fractions
  v. substitution to integrate expressions
  vi. linear substitution
  vii. integration by parts
  viii. differentiating and then integrating
- First order linear differentiation
- Applications of differential equations, but not including input/output problems

Examples of learning activities:

- In class activities – including individual and group activities to develop conceptual understanding and skill development. Involving such questions as:

  Determine $\int \sin^3 x \, dx$

  Use partial fractions to determine $\int \frac{x+3}{x(x+1)} dx$

  By using an appropriate trigonometric identity obtian the exact value of $\int_0^1 \tan^2\left(\frac{\pi x}{4}\right) dx$

  Further activities can be found in *Activities for Pairs and/or Groups*

- Assignments – covering the concepts and applying that learning through short answer and extended answer questions. Using problem solving strategies to determine the answer and discuss the appropriateness of the answer

- Unit test – Developing time management skills and to solve a range of problems within the topic without guidance

- Application
  i. Newton's law of cooling (experiment)
  ii. Making coffee using coffee filter machine
  iii. Logistics equations in business and economic

Learners are required to investigate a minimum of two applications over the course of the subject, from at least two of the units

Activities for Individuals, Pairs and Groups

The activities in this section are not necessarily in the order in which they should be done. They are intended to give ideas and to be adapted to suit the needs of different learners rather than be used exactly as written. Several activities may cover the same objective – this is to give choice rather than repetition. Activities are not intended to cover every aspect of the scheme of work. They are opportunities for a more interactive way of learning where appropriate.

Most activities are more effective if done in pairs or groups.

At the end of each activity, particularly matching ones, learners should be encouraged to write up one or two of the examples with detailed reasons for future reference.

Unit 1 – Sequences and Series

Activity 1

The headquarters of *Spires International* resides in a skyscraper that is 50 floors high and is covered entirely by windows on all four sides. Each floor has 38 windows. Once a year, all the windows are cleaned on the outside. The cost of cleaning the windows is:

- $2.00 for each first floor window
- $2.50 for each second floor window
- $3.00 for each third floor window, and so on

How much will it cost to clean the windows of this building? What if the building is 103 floors high? What if the building is n floors tall?

Once learners have solved this problem they are to come up with a similar problem and exchange with another learner in the class.

Activity 2

This is a group activity and each group is to be given a *Comment on the Validity* - Work-sheet. Through discussion within the group, comment on the validity of each statement and explain your reasoning.

Activity 3

Match up the sigma expressions that are equivalent. Although learners may well do this initially by substituting numbers they must justify their decisions algebraically as well.

Give each pair of learners an expression such as $\sum_{r=1}^{7}(r-3)^2$ and ask them to write it in the middle of a large piece of paper. Then ask them to see how many different ways they can write it. Pass it to another pair for checking and then justify some algebraically.

Activity 2 – *Comment on the Validity* - Work-sheet

Example A

Find the sum of this infinite geometric series:

$$20.4, -5.1, 1.275, \ldots$$

It can be solved like this:

First we find the common ratio:

$$r = \frac{-5.1}{20.4} = \frac{1.275}{-5.1} = -0.25$$

This is reasonable value for r since we know the series converges, so we must have $-1 < r < 1$ and – 0.25 is contained in this interval so we have: $u_1 = 20.4, r = -0.25$

$$S_\infty = \frac{u_1}{1-r} = \frac{20.4}{1-0.25} = 27.2$$

We can check this with the calculator/spreadsheet by using the formula for $S_n = \frac{u_1(1-r^n)}{1-r}$ and creating a table of values.

Comment on the validity of the statement below and explain your reasoning.

A	B
1	34.0000000000
2	25.5000000000
3	27.6250000000
4	27.0937500000
5	27.2265625000
6	27.1933593750
7	27.2016601563
8	27.1995849609
9	27.2001037598
10	27.1999740601
11	27.2000064850
12	27.1999983788
13	27.2000004053
14	27.1999998987
15	27.2000000253
16	27.1999999937

We can see that the value of S_n oscillates around 27.2, getting closer and closer to 27.2 as the value of n increases.

Example 2

For a particular series:

$$S_n = \frac{27n + 3n^2}{2}$$

Find u_n and **prove** that the series is arithmetic.

It can be solved like this:

For any series, $u_n = S_n - S_{n-1}$

$$S_n = \frac{27n + 3n^2}{2} \rightarrow S_{n-1} = \frac{27(n-1) + 3(n-1)^2}{2}$$

$$\therefore u_n = S_n - S_{n-1} = \frac{27n + 3n^2}{2} - \frac{27(n-1) + 3(n-1)^2}{2}$$

$$\therefore u_n = \frac{27n + 3n^2 - 27n + 27 - 3(n-1)^2}{2}$$

$$\therefore u_n = \frac{27n + 3n^2 - 27n + 27 - 3n^2 + 6n - 3}{2}$$

$$\therefore u_n = \frac{24 + 6n}{2}$$

$$\therefore u_n = 12 + 3n$$

$$\therefore u_n = 12 + 3n \rightarrow u_{n+1} = 12 + 3(n+1)$$

$$\therefore u_{n+1} - u_n = 15 + 3n - 12 - 3n = 3$$

Comment on the validity of the statement below and explain your reasoning.

Since the difference between any term in the series and the term before it is a constant value, the series must be arithmetic (with a common difference of 3).

Activity 3 – *Matching up sigma expressions* work-sheet

$\sum_{r=0}^{9} (r+1)(r+2)$
$\sum_{r=1}^{10} r(r-1)$
$\sum_{r=2}^{11} (r^2 - r)$
$\sum_{r=0}^{9} (r^2 + r)$
$\sum_{r=2}^{11} (r-1)(r-2)$
$\sum_{r=21}^{30} (r-20)(r-19)$
$\sum_{r=1}^{10} r(r-1)$
$\sum_{r=21}^{30} (r-20)(r-21)$

Unit 2 – Complex Numbers

Activity 1

Ask learners to investigate:

$$(1+i)^n \; for\ n = 1, 2, 3, etc\ and\ express\ your\ answer\ in\ the\ form\ of\ a + bi$$

Activity 2

Ask learners to find the errors in the following: (Give the correct answer as part of your working)

Question 1 Solve $2z + i = 3 - z$

Solution:

Put: $z = x + yi$ into the original equation

Then: $2(x + yi) + i = 3 - (x + yi)$

$\therefore$ $2x + 2yi + i = 3 - x - yi$

So $2x = 3 - x$ and $2y = -y$

This gives $x = 1$ and $y = 0$

Therefore: $z = 1$

Question 2 Simplify $i(3 - 4) + (2 - i)(4i + 2)$

Solution:

$$i(3-4) + (2-i)(4i+2) = 3i - 4 + 8i - 4 + 4 - 2i$$

$$i(3-4) + (2-i)(4i+2) = 9i - 4$$

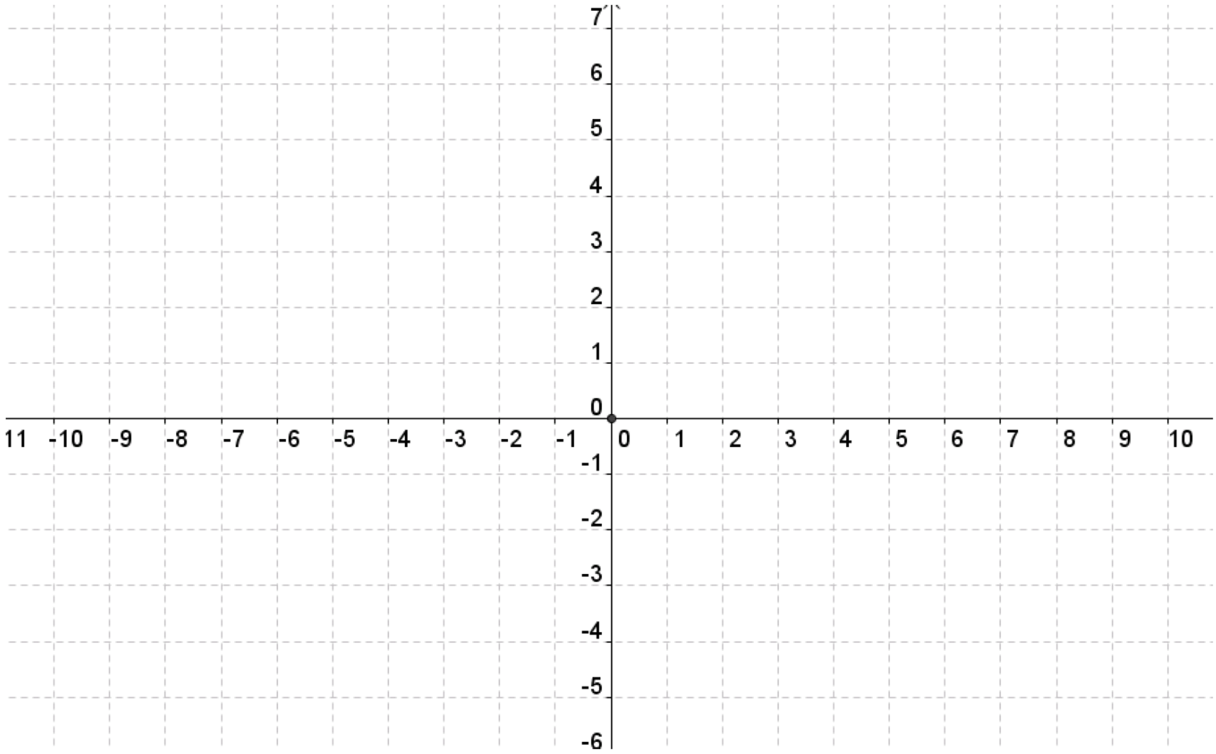
Activity 3

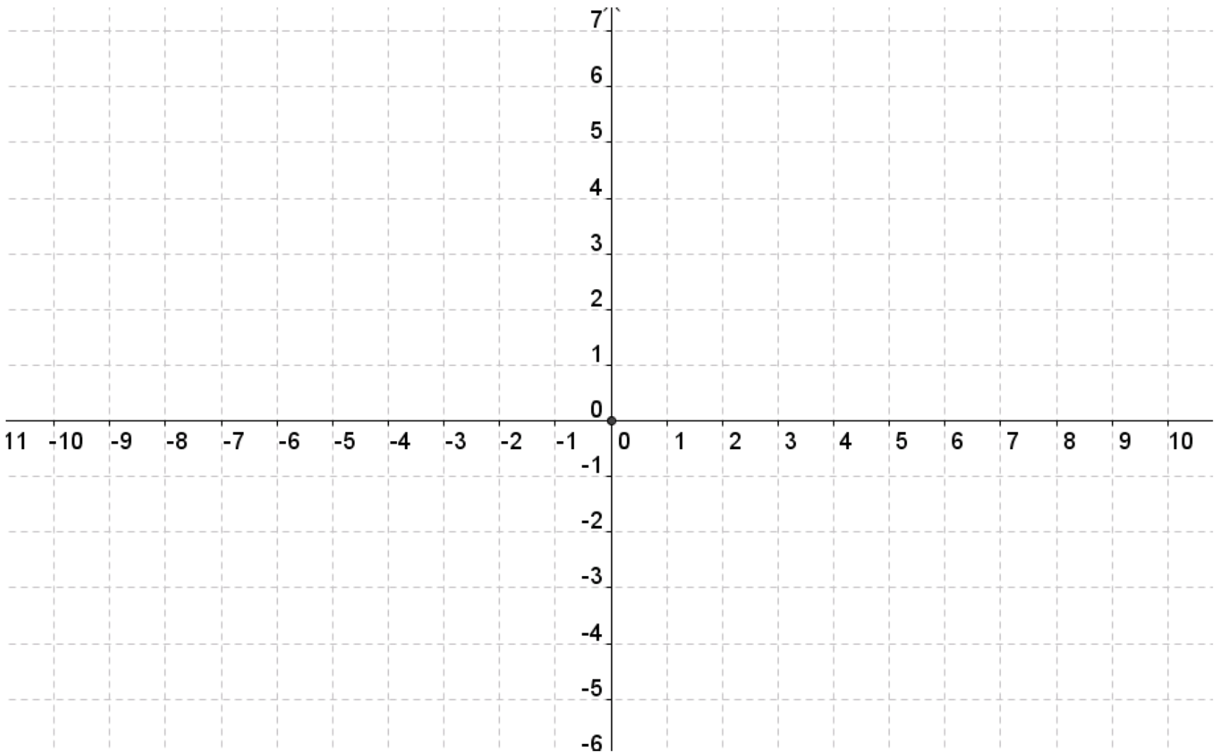
Place the following values on the Argand Plane (Argand Plane activity sheet) and calculate the modulus in each case.

	Complex Number	Modulus |z|
a	$z_1 = 3 + 2i$	
b	$z_2 = 2 + 3i$	
c	$z_3 = -3 + 4i$	
d	$z_4 = -2 - 4i$	
e	$z_5 = i$	
f	$z_6 = -2 - i$	
g	$z_7 = 1 + \sqrt{25}i$	

- write in your own words, what is meant by the modulus of a complex number.
- what is the mathematical notation for the modulus of a complex number z?
- list four complex numbers with a modulus equal to 4.
- is $|z_1 + z_2|$ equal to $|z_1 - z_2|$? always? sometimes? never? explain using examples plotted on the argand plane
- what do you notice about $|a + bi|$ and $|a - bi|$?
- describe how you could show all the complex numbers that have a modulus of 3.
- what do all the numbers on the real axis of the argand plane have in common?
- what do all the numbers on the imaginary axis of the argand plane have in common?

Activity 3 – Argand Plane grids





Activity 4

Complex Numbers Battle ships

$(4-i)+(3+2i)$	$(5-8i)^2$	$(2+3i)+(7+i)$	$8i(2-i)$	$(8+5i)-(1+2i)$	$(3+i)(3-i)$	$\frac{2i}{3i}$	$\lvert 4i\rvert$
$\frac{3+3i}{2-i}$	$(7+5i)-(1+5i)$	$(5+3i)(1-4i)$	$\lvert 1+4i\rvert$	$\frac{6+4i}{1-2i}$	$(4+9i)+(-6+i)$	$\lvert 2+4i\rvert$	$(6+2i)-(5-i)$
$5i(-2+i)$	$\lvert 2+6i\rvert$	$\frac{5+3i}{i}$	$(4+7i)+(4-7i)$	$-2i(6-5i)$	$\frac{4+3i}{4-2i}$	$(1-5i)(2+i)$	$4i(6-i)$
$(5+8i)(2-i)$	$(6+3i)(6-3i)$	$6-(2+9i)$	$(2-i)-(3+2i)$	$5i(7-4i)$	$\lvert 1+3i\rvert$	$(-1-i)+(8-2i)$	$\frac{5+i}{1-i}$
$(7-4i)(-1+2i)$	$\lvert 5i\rvert$	$\lvert 3+2i\rvert$	$\frac{1+2i}{5-3i}$	$\lvert -1+5i\rvert$	$(9+5i)(3+2i)$	$\frac{8-2i}{1+i}$	$\lvert -7i\rvert$
$\frac{5+3i}{1-2i}$	$\frac{3i}{4-2i}$	$(5+i)-(8+i)$	$i(3+i)$	$\lvert 2+6i\rvert$	$(3+10i)^2$	$(5+i)+(8+i)$	$\lvert 3-2i\rvert$
$(2+5i)(1-2i)$	$\lvert -2i\rvert$	$-2i(4+i)$	$\lvert 8+6i\rvert$	$4i(-2-3i)$	$\lvert 3i\rvert$	$\frac{8+i}{1-i}$	$(2+i)(1-i)$
$\lvert 3+4i\rvert$	$(5+i)(3-5i)$	$(3-i)-(8+6i)-3i$	$\lvert 3-4i\rvert$	$\frac{2+3i}{5-2i}$	$(2-3i)(2-i)$	$(2-i)-(-4+i)$	$(12+i)-(8+i)$
$(-6+i)(3-i)$	$\lvert 3-5i\rvert$	$\frac{2+3i}{-i}$	$(6+2i)-(6+i)$	$\lvert 5-3i\rvert$	$\frac{5+i}{-2i}$	$(3+i)-(3-i)$	$-10i(4+7i)$
$\frac{2-3i}{1+2i}$	$\lvert 2+3i\rvert$	$(5+i)-(5+i)$	$(2-9i)(9-6i)$	$\frac{-1-2i}{1+2i}$	$(25+i)-(25-3i)$	$(5+4i)(5-4i)$	$\frac{7-2i}{3-i}$

Give each learner two laminated copies of the game-board (one on white card one on coloured card) and erasable whiteboards markers

Rules
This game is played like Battleships, except instead of placing ships each player marks squares on his/her white game board as follows:

- Each block of squares must be either horizontal or vertical (no diagonals or turns allowed)
- Each player must mark one (1) single, two (2) double, two (2) triple and one (1) quadruple block of squares
- No blocks may overlap, though they can be adjacent

The coloured game-board is for marking the squares a player has guessed and the white game-board is for marking the squares his/her opponent has guessed.

Players take turns in guessing an opponent's square, but must use correct complex form to make their guess – working out paper is allowed.

The game is over when one player finds all of her/his opponents blocks.

Complex Numbers game-board

$(4-i)+(3+2i)$	$(5-8i)^2$	$(2+3i)+(7+i)$	$8i(2-i)$	$(8+5i)-(1+2i)$	$(3+i)(3-i)$	$\frac{2i}{3i}$	$\lvert 4i\rvert$
$\frac{3+3i}{2-i}$	$(7+5i)-(1+5i)$	$(5+3i)(1-4i)$	$\lvert 1+4i\rvert$	$\frac{6+4i}{1-2i}$	$(4+9i)+(-6+i)$	$\lvert 2+4i\rvert$	$(6+2i)-(5-i)$
$5i(-2+i)$	$\lvert 2+6i\rvert$	$\frac{5+3i}{i}$	$(4+7i)+(4-7i)$	$-2i(6-5i)$	$\frac{4+3i}{4-2i}$	$(1-5i)(2+i)$	$4i(6-i)$
$(5+8i)(2-i)$	$(6+3i)(6-3i)$	$6-(2+9i)$	$(2-i)-(3+2i)$	$5i(7-4i)$	$\lvert 1+3i\rvert$	$(-1-i)+(8-2i)$	$\frac{5+i}{1-i}$
$(7-4i)(-1+2i)$	$\lvert 5i\rvert$	$\lvert 3+2i\rvert$	$\frac{1+2i}{5-3i}$	$\lvert -1+5i\rvert$	$(9+5i)(3+2i)$	$\frac{8-2i}{1+i}$	$\lvert -7i\rvert$
$\frac{5+3i}{1-2i}$	$\frac{3i}{4-2i}$	$(5+i)-(8+i)$	$i(3+i)$	$\lvert 2+6i\rvert$	$(3+10i)^2$	$(5+i)+(8+i)$	$\lvert 3-2i\rvert$
$(2+5i)(1-2i)$	$\lvert -2i\rvert$	$-2i(4+i)$	$\lvert 8+6i\rvert$	$4i(-2-3i)$	$\lvert 3i\rvert$	$\frac{8+i}{1-i}$	$(2+i)(1-i)$
$\lvert 3+4i\rvert$	$(5+i)(3-5i)$	$(3-i)-(8+6i)-3i$	$\lvert 3-4i\rvert$	$\frac{2+3i}{5-2i}$	$(2-3i)(2-i)$	$(2-i)-(-4+i)$	$(12+i)-(8+i)$
$(-6+i)(3-i)$	$\lvert 3-5i\rvert$	$\frac{2+3i}{-i}$	$(6+2i)-(6+i)$	$\lvert 5-3i\rvert$	$\frac{5+i}{-2i}$	$(3+i)-(3-i)$	$-10i(4+7i)$
$\frac{2-3i}{1+2i}$	$\lvert 2+3i\rvert$	$(5+i)-(5+i)$	$(2-9i)(9-6i)$	$\frac{-1-2i}{1+2i}$	$(25+i)-(25-3i)$	$(5+4i)(5-4i)$	$\frac{7-2i}{3-i}$

Complex number answers to game-board

$7+i$	$-39-80i$	$9+4i$	$8+16i$	$7+3i$	10	$\frac{2}{3}$	4
$\frac{3+9i}{5}$	6	$17-17i$	$\sqrt{17}$	$14-8i$	$-2+10i$	$\sqrt{20}$	$1+i$
$-5-10i$	$2\sqrt{10}$	$3-5i$	8	$-10-12i$	$\frac{1+2i}{2}$	$7-9i$	$4+24i$
$18-3i$	45	$4-9i$	$-1-3i$	$20+35i$	$\sqrt{10}$	$7-3i$	$2+3i$
$1+18i$	5	$\sqrt{13}$	$\frac{-1+13i}{34}$	$\sqrt{26}$	$17+33i$	$6-10i$	7
$-\frac{1}{5}+\frac{13}{5}i$	$\frac{-6+7i}{20}$	-3	$-1+3i$	$2\sqrt{10}$	$-91+60i$	$13+2i$	$\sqrt{13}$
$12+i$	2	$2-8i$	10	$12-8i$	3	$\frac{7-9i}{2}$	$3-i$
5	$20-22i$	$-5+2i$	5	$\frac{4+19i}{29}$	$1-6i$	$6-5i$	4
$-17+9i$	$\sqrt{33}$	$3+2i$	i	$\sqrt{34}$	$\frac{-1+5i}{2}$	$2i$	$70-40i$
$\frac{-4-7i}{5}$	$\sqrt{13}$	0	$-36-93i$	-1	$4i$	9	$\frac{23+i}{10}$

Unit 3 – Matrices and Linear Algebra

Activity 1
Using the matrices given on the sheet, ask learners which they can combine and in what order, to make the following transformations:

Rotation of 180°

Identity

Reflection in the *x*-axis

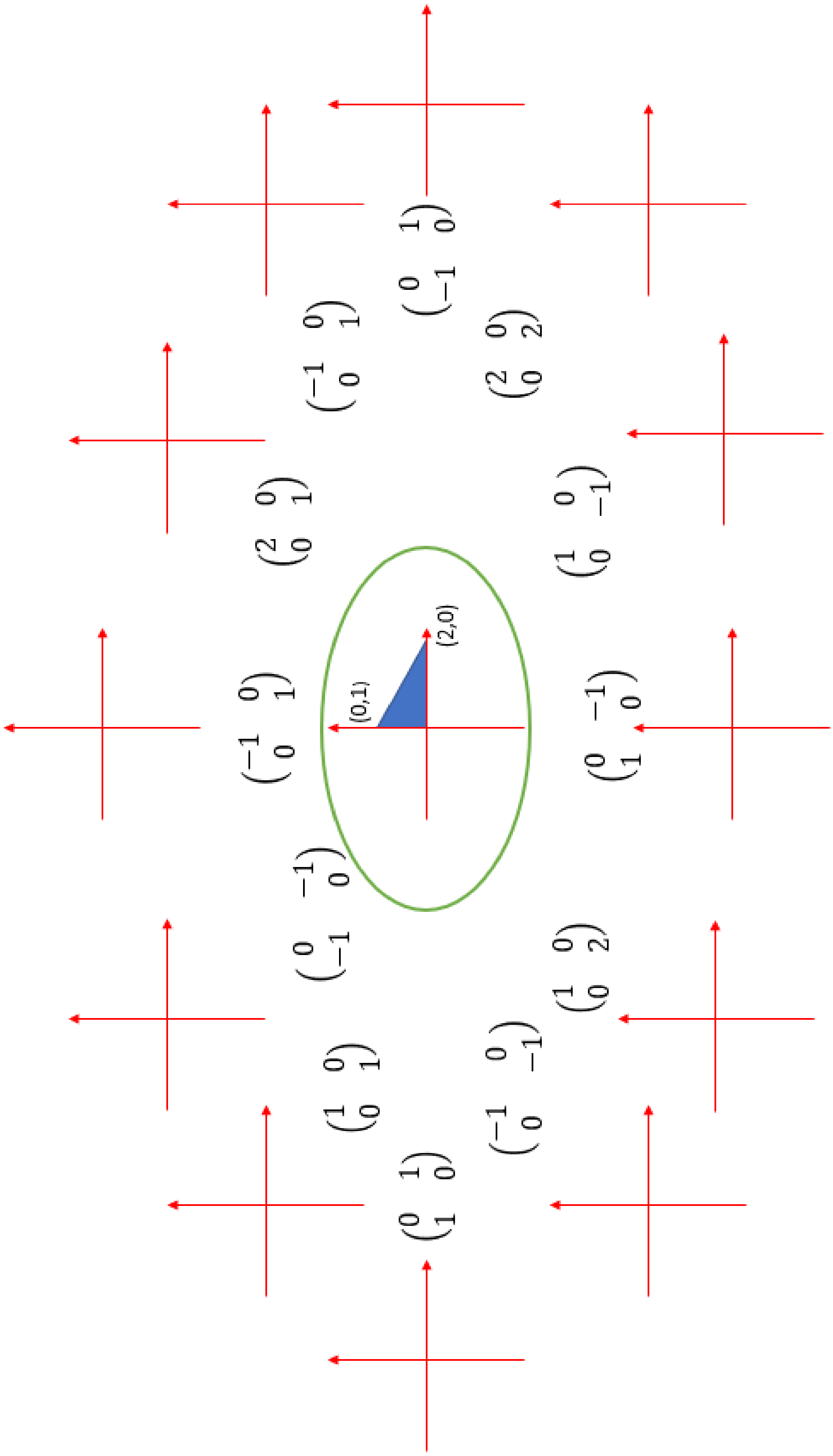
Reflection in the line $y = -x$.

Ask them if they can make any of them in more than one way. They should justify their answers using matrix multiplication.

Activity 2
Give each pair of learners a copy of the sheet enlarged onto A3. Learners have to multiply the matrix by each position vector of the triangle, plot the new position of the triangle and state the transformation that the matrix represents. This enables learners to find out which transformation each matrix represents.

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

Activity 2 – photocopy to A3



Unit 4a – Differentiation and integration to find volume of definite 3D shapes

Activity 1

Cut the sheet "Whispers" into separate cards and number them as 1.1, 2.1, 3.1 ….. Then a learner picks up one of them say 2.1. They solve the differential equation and write their solution on another piece of card numbered 2.2. Another learner then picks up 2.2 and differentiates the solution putting this on a new piece of card numbered 2.3. This continues for as long as you want. At the end each set is collated and the sequence is examined by the class. If the start is the same as the beginning they leave it but if it is not, the class find where the error occurred and what it was.

Activity 2

On the sheet "True or False" learners should decide which cards are true and which are false. They should also state what is wrong with the false ones.

Activity 3

Starting with each card on the sheet "Formulating Differential Equations", learners should write a question that will result in the information on the card. They can check by giving their question to another learner who is doing a different card and then seeing if each gets the same differential equation. They could go on to solving the equations.

Activity 4

Challenge learners to use the volume of integration formula to check the standard formulas for the volume of a sphere and the volume of a cone.

$\frac{dy}{dx} = \frac{x^2}{y}$	$\frac{dy}{dx} = \frac{3}{yx^2}$
$\frac{dy}{dx} = \frac{1-x}{e^{3y}}$	$\frac{dy}{dx} = y\sin x$
$\frac{dy}{dx} = \frac{1}{(y+1)(x+1)}$	$\frac{dy}{dx} = \frac{x+3}{y-2}$
$\frac{dy}{dx} = \frac{x^2+1}{y}$	$\frac{dy}{dx} = xe^y$
$\frac{dy}{dx} = \frac{(x+3)^4}{\cos y}$	$\frac{dy}{dx} = \frac{x-1}{y+2}$

$\frac{dy}{dx} = x + 1 \Rightarrow y = \frac{x^2}{2} + x + c$
$\frac{dy}{dx} = xy \Rightarrow y = \frac{x^2}{2} \times \frac{y^2}{2} + c$
$\frac{dy}{dx} = \frac{x}{y}$ and $x = 0, y = 0 \Rightarrow x^2 = y^2$
$\frac{dP}{dt} = 3t \Rightarrow P = Ae^{3t}$
$\frac{dM}{dt} = -2M \Rightarrow M = Ae^{-2t}$
$\frac{dA}{dr} = 2r$ and $\frac{dV}{dA} = 3 \Rightarrow \frac{dV}{dr} = 6r$

$\frac{dm}{dt} \propto m$ $m=3g$ at $t=0$ $m=7g$ at $t=2s$	
$\frac{dh}{dt} = -\frac{k}{h}$ k constant $h = 60\,cm$ at $t = 0$ $h = 20\,cm$ at $t = 3\,s$	
$\frac{dV}{dr} = 4\pi r^2$ $\frac{dr}{dt} = 4cms^{-1}$	
$\frac{ds}{dt} \propto s^2$ $s=4m$ at $t=6\min$ $s=11m$ at $t=7\min$	
$\frac{dP}{dt}=4P,$ $P=25$ at $t=0$	

Unit 4b – Integration

Activity 1
Learners should each take a pink (pale red), a blue card, a green card and a yellow card. On the relevant coloured card write a function that:

- Pink card – that requires the parts to be integrated
- Blue card – that requires substitution to integrate it
- Green card – that requires partial fractions to integrate it
- Yellow card – one that requires NONE of the following, basic rules or simple reverse chain rule only.

They put their name on each card and take a card of each colour written by someone else. They then do the integration and return to the author for marking. If there are any queries about the appropriateness of a function for that category that should be referred back to the author for discussion.

Activity 2
This is all about choosing a suitable method for integration rather than doing the actual integration. Using the sheet "Methods of Integration" learners should cut them all out, then draw a suitable Venn diagram on a large piece of paper and sort the integrations into the categories "Partial Fractions", Substitution", "Parts" and "Basics / Reverse Chain Rule" and stick them onto the Venn diagram. There should be lots of discussion about which ones can go in more than one set and about the one that does not fit anywhere.
Learners can always add more of their own and could choose one of each to write about. This would include writing as to why it goes in that category and how to do the integration.

Activity 3
Sheet "True or False?" contains a mixture of integration and differentiation cards. Each one has to be categorised as true or false along with a reason.

Activity 4
Learners should integrate functions such as $\int x(x+3)^6\,dx$ using both parts and substitution to see that they give the same answer – good for discussion as initially they appear to be different but after some algebraic manipulation they both give the same answer.

Activity 5
The sheet "Homework" has to be marked and comments written on to help the learner improve their integration

$\int \frac{x^2}{\sqrt{x^3+1}}dx$	$\int \frac{x}{(x+2)^2}dx$	$\int 4\sin\frac{1}{2}x\,dx$
$\int \frac{x^2-1}{x}dx$	$\int \frac{dx}{x^5}$	$\int x(x+7)^4\,dx$
$\int \frac{dx}{2x+7}$	$\int x^2\sin x^3\,dx$	$\int x\cos 5x\,dx$
$\int \frac{2x-1}{x^2-x-6}dx$	$\int \frac{x}{x^2-4}dx$	$\int \sin x\cos^2 x\,dx$
$\int x\sqrt{x^2+2}\,dx$	$\int x\cos^2 x\,dx$	$\int x\sqrt{x+5}\,dx$
$\int \frac{7}{x^2}dx$	$\int (x+1)(x+3)^5\,dx$	$\int \sin^6 x\cos x\,dx$
$\int \frac{4}{x^2+7x+12}dx$	$\int \frac{2x-1}{x+4}dx$	$\int \sin x\cos x\,dx$
$\int \frac{3}{x^2-x-2}dx$	$\int \frac{1}{(2x+3)^4}dx$	$\int \sqrt{2x-1}\,dx$
$\int \frac{x}{x+1}dx$	$\int \frac{4}{3x-1}dx$	$\int xe^{2x}\,dx$
$\int \frac{4}{e^{3x}}dx$	$\int \frac{2x-1}{x^2+2x-1}dx$	$\int e^{2x+3}\,dx$
$\int \frac{x}{x^2-4}dx$	$\int (x+2)^6\,dx$	$\int x\sin x\,dx$

\

$\frac{d}{dx}\left(\sin^4 x\right) = 4\cos^3 x$
$\frac{d}{dx}\left(xe^x\right) = xe^x + e^x$
$\int \sin 4x\,dx = -\tfrac{1}{4}\cos 4x + c$
$\int (3x-1)^7\,dx = \frac{(3x-1)^8}{24} + c$
$\int \sin^2 x\,dx = \frac{\sin^3 x}{3} + c$
$\int \frac{3}{7-x}\,dx = 3\ln\lvert 7-x\rvert + c$
$\frac{d}{dx}\left(\tan 2x\right) = \sec^2 2x$
$\int x\left(1+\frac{1}{x}\right)dx = \frac{x^2}{2} + \ln x + c$
$\int \sin x\cos^5 x\,dx = \frac{\cos^6 x}{6} + c$
$\frac{d}{dx}\left(\ln(2x-1)\right) = \frac{2}{2x-1}$
$\frac{d}{dx}\left(\sin x\cos x\right) = \cos 2x$
$\int \cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin 2x + c$

Activity 5 *Homework* work-sheet

1) $\int \frac{3}{4x-2}dx = 3\ln(4x-2)+c$

2) $\int \frac{4dx}{\sqrt{3-2x}} = 4\ln\sqrt{3-2x}+c$

3) $\int 4\cos(3t-1)dt = -\tfrac{4}{3}\sin(3t-1)+c$

4) $\int (7x+3)^5\,dx = \int \frac{(7x+3)^6}{6}$

5) $\int 3e^{2x+1}dx = 6e^{2x+1}+c$

6) $\int \cos^2 x dx = \tfrac{1}{3}\cos^3 x + c$

7) $\int e^{3x}dx = \tfrac{1}{4}e^{4x}+c$

8) $\int \sin 2x dx = -\tfrac{1}{2}\cos 2x + c$

9) $\int xe^{x^2}dx = \tfrac{1}{2}e^{x^2}+c$

10) $\int \sec^2 2x dx = \tan 2x + c$

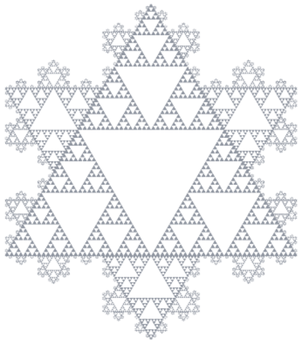
EXAMPLES OF APPLICATIONS

Applications are investigations that are designed to apply the content knowledge to a real-world scenario through problem solving. Applications may be contained within a lesson; multiple lessons or a combination of lesson time and homework.

The following applications have been provided by teachers and colleges and have been acknowledged for their contribution at the end of the application.

- Koch Snowflake
- Volume of a Bottle
- The Mandelbrot Set
- Zeros of Cubic Functions
- Area of Approximation Investigation
- Cryptography
- Newton's Law of Cooling
- Markov Chain

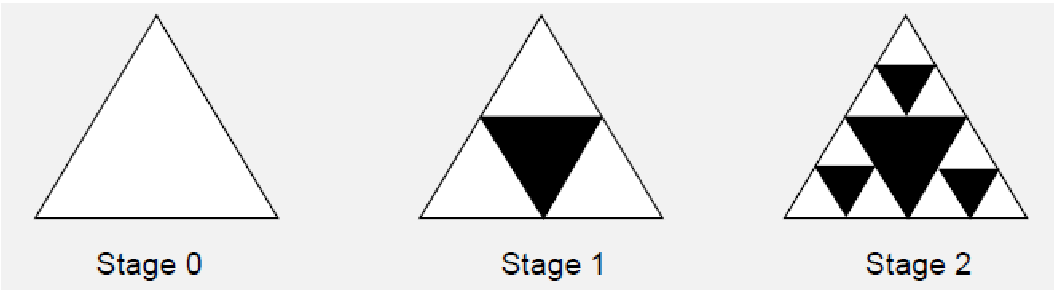
Koch Snowflake



Introduction
The Koch snowflake (also known as the Koch curve, Koch star or Koch island) is a mathematical curve and one of the earliest fractal curves to have been described. It first appeared in a paper by the Swedish mathematician Helge von Koch in 1904 titled "On a continuous curve without tangents, constructible from elementary geometry".

The progression for the area of the snowflake converges to $\frac{8}{5}$ times the area of the original triangle, while the progression for the snowflake's perimeter diverges to infinity, i.e. the snowflake has a finite area bounded by an infinitely long line.

Section 1
Begin with an equilateral triangle and put a black triangle half the scale and upside down inside the original triangle, resulting in 3 white triangles and 1 black triangle. Continue doing this indefinitely, the figures below illustrate the initial stages.



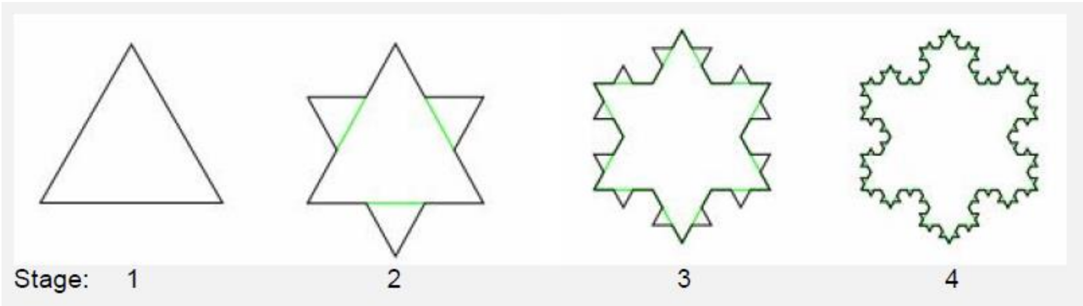
1. Find the area of the first black triangle if the area of the main triangle is 1 unit^2.
2. Determine the total area of the black triangles added at Stage 2.
3. Express your answer to the above question as a fraction of the area of the first black triangle.
4. Explain why the area of the triangles added at the n^{th} stage is the same fraction of the area of the triangles added at the $(n-1)^{th}$ stage.
5. Write down an expression for the area of all the black triangles added at the first n stages and then determine the area as $n \to \infty$.
6. By observing the white triangles determine the fraction of the main triangle that is still white at Stage 1, Stage 2, Stage 3 and son on to Stage n.
7. Comment on the area of the white triangles as $n \to \infty$.

Section 2

The complete Koch Snowflake can be constructed by starting with an equilateral triangle, then recursively altering each line segment as follows:

1. Divide the line segment into three equal lengths
2. Draw an equilateral triangle that has the middle segment from step 1 as its base and points outwards
3. Remove the line segment that is the base of the triangle from step 2 (after one iteration of this process the resulting shape is the outline of a hexagram

The Koch snowflake is the limit approached as the above steps are continually repeated on each new triangle (see diagram). The Koch curve originally described by von Koch is constructed with only one of the three sides of the original triangle. In other words, three Koch curves make a snowflake.



4. If the total area added on when the Koch snowflake is developed indefinitely, show that it results in a finite area equal to $\frac{8}{5}$ of the area of the initial triangle.
5. Show that the Koch snowflake has a perimeter of an infinite length if the process is continued indefinitely.

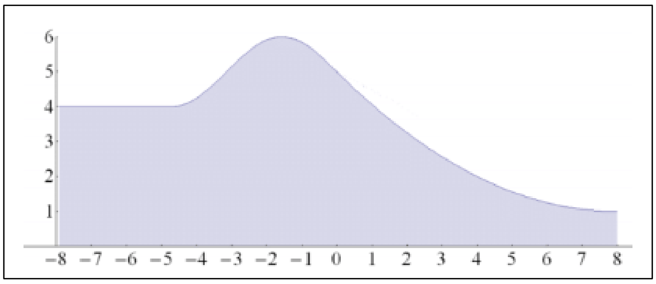
This application was provided by Adrian Baron and used with his consent.

Volume of a Bottle

You are to design a fancy wine bottle using your knowledge of integration and differentiation. The fancy wine bottle is to be made by rotating a function around an axis.

The function describing your wine bottle is to be made up of at least three (3) different functions over different domains.

$$y = 4; \quad -7.92 \le x \le -\frac{3\pi}{2}$$
$$y = 5 - sinx; \quad -\frac{3\pi}{2} < x \le 0$$
$$y = \frac{1}{16}x^2 - x + 5; \quad 0 < x \le 8$$



Your bottle is to have the following properties:
(all demonstrated in the example above)

- made of at least:
  - three (3) different functions over different domains
  - two (2) non-linear functions
  - one (1) function non-polynomic (counts towards non-linear functions)
- continuous – your functions must be equal at the edges of the domains
- no sharp corners – the first derivative of your functions must be equal at the edges of the domains
- your bottle must be parallel to axis of rotation at the two furthest ends
- the volume of your bottle (the volume of the revolution) must be 750. (Values between 749.5 and 750.5, the example above is 750.15)
- your bottle must be bottle shaped, i.e. should have a thin neck and have a reasonable sized base to stand on. Artistic wiggles are fine and are expected.

Your report is to include:

- a list of your functions and their domains
- a neat graph of your combined function with points at the edge of domains labelled
- calculations showing that your bottle is:
  - continuous
  - free of sharp corners at each domain edge
  - parallel to the axis of rotation at its two ends
- calculations determining the volume of your bottle

Demonstrating a greater expertise and understanding of integral and differential calculus will achieve higher results. Some possible ways could include:

- use of more than three (3) functions
- use f difficult or unusual non-polynomic functions(s)
- non-stationary edges to domains
- exact values in all calculations

This application was provided by Ian Miller and used with his consent.

The Mandelbrot Set

Initial questions (slide 1)

- What is the Mandelbrot Set?
- How does it relate to complex numbers?
- Can we use the calculator to investigate the Mandelbrot set?

Introduction (slide 2)

The Mandelbrot set was discovered in 1980 by Benoit Mandelbrot. The set is produced by the incredibly simple iteration of the formula:

$$Z_{n+1} = Z_n^2 + c, where\ Z\ and\ c\ are\ complex\ numbers\ and\ Z_0 = 0$$

This can be written without complex numbers as:

$$x_{n+1} = x_n^2 - y_n^2 + a$$

$$y_{n+1} = 2x_n y_n + b$$

where $Z = (x, y)$ and $c = (a, b)$

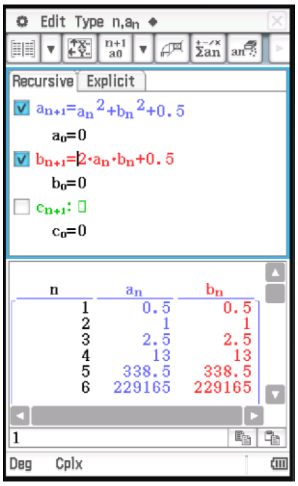
In or out of the set! (slide 3)

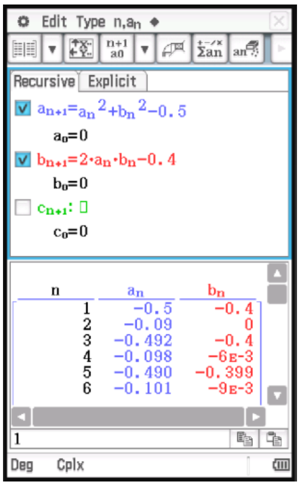
- If a number does not diverge and stays within a closed region it will be part of the Mandelbrot set and will be black
- If it diverges then the number of iterations when it is clear that it is divergent will be indicated by a certain colour.

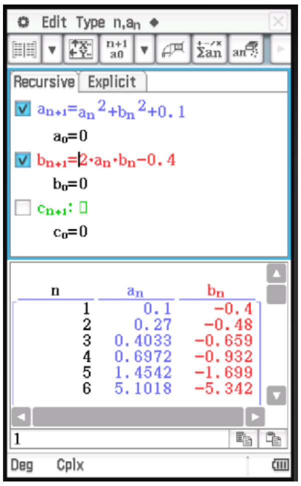
Entering the ClassPad (slide 4)

- We can then enter in into the ClassPad in the Sequence mode and consider a lot of the numbers and check if they are IN the set or NOT IN the set
- There are a lot of calculations to do and we will just look at some such as:
    - $c = 0.5 + 0.5i$
    - $c = -0.5 - 0.4i$
    - $c = 0.1 - 0.4i$

ClassPad and Sequences (slides 5 and 6)







What is the pattern? (slide 7)

- Look at some points for c?
- What values could you rule out as definitely not being a part of the Mandelbrot set?

The Mandelbrot set (slide 8)

- The whole Mandelbrot set lies within a circle of radius 2.5 with the centre the origin of the complex plane
- It is certainly skewed towards the negative real part of the axis

Divergence or not? (slide 9)

- At locations where divergence is indicated the point is displayed according to a scale that represents how many iterations are needed to show divergence
- If the pixel rapidly converges or does not diverge for a large number of iterations the point will be black

Image (slide 10)

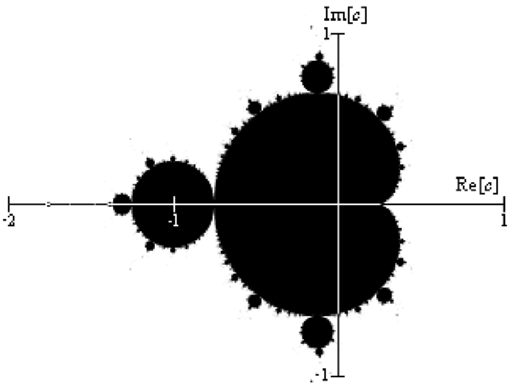


Image (slide 11)

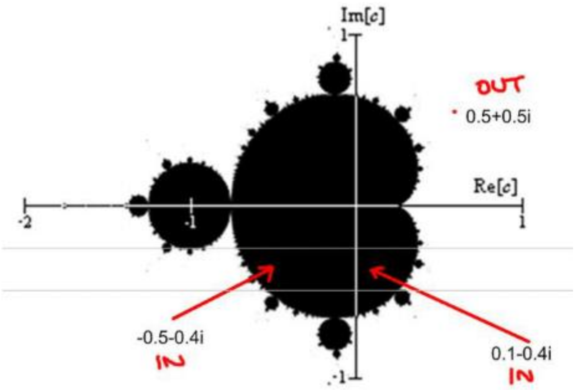
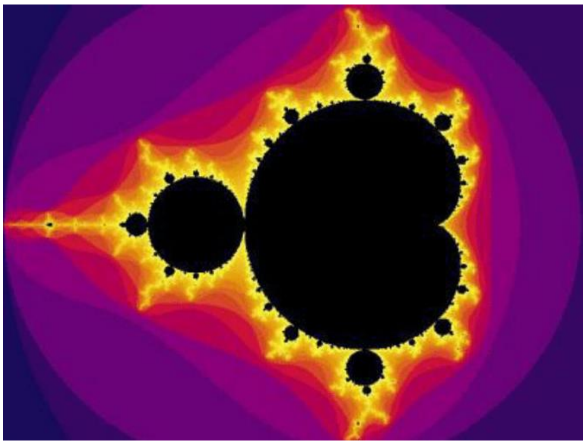


Image (slide 12)



Mandelbrot set shape (slide 13)

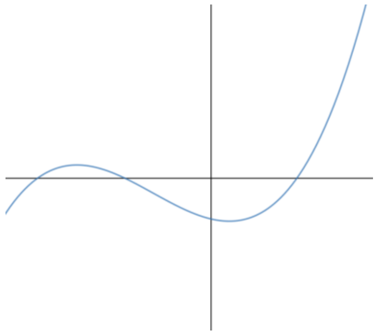
- The set is described as a cardioid with a circle to the left and some warts above and below which add to the beauty (or ugliness) of the graph
- Zooming in near the boundary produces other unusual patterns

References (slide 14)

http://www.daviddarling.info/encyclopedia/M/Mandelbrot_set.html
This is IT, Forster, Cadby & Young, AAMT, 1998

This application was prepared as a PPT by Brett Stephenson and used with his consent.

Zeros of Cubic Functions



Part 1
Consider the cubic function:

$$f(x) = 2x^3 + 6x^2 - 4.5x - 13.5$$

Find the roots of the cubic.
Taking the roots two at a time, find the equations of the tangents at the points on the curve where the values of x are the average of the values for the two roots. (There will be three such tangents)

Find where the tangents meet the curve again. Comment on the results!

Part 2
State a conjecture concerning the roots of a cubic and the tangents at these average values.

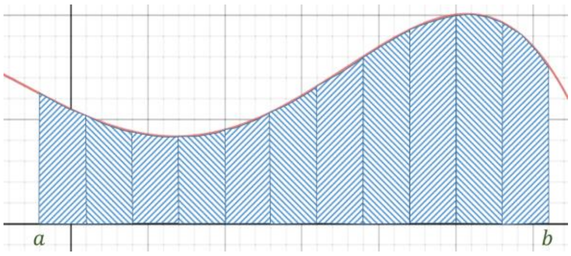
Test your conjecture on other similar cubic functions. Is the result still true for cubics with repeated roots?

Part 3
Prove your conjecture holds for any cubic with three real roots.

Investigate what happens for a cubic with one real and two complex roots.

This application was provided by Julia Waterworth and used with her consent.

Area Approximation Investigation



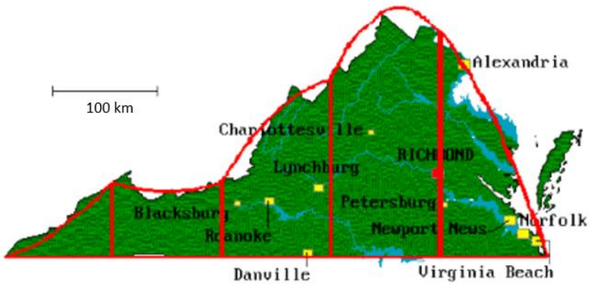
Task 1 – Research

Research the following terms and rules used to approximate areas:

- Reimann Sums
- Right Triangles
- Left Triangles
- Midpoints
- Trapezoidal Rule
- Simpson's Rule

Task 2 – Application

Provide a specific example which illustrates each of these techniques and compares approximations of the same area calculated with the exact value found using the fundamental theorem of calculus.



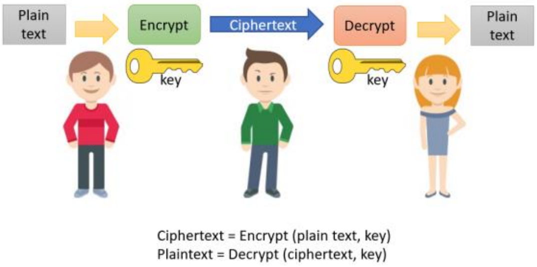
This application was provided by Hellyer College and used with their consent.

Cryptography

Cryptography is the study of codes. Codes are used, for example, by the military as well as for the protection of credit card numbers and to make sure that the process of purchasing goods over the internet is secure.

One of the earliest attempts to use Matrices to hide information was developed by Lester Hill in the 1920's.

Task 1
Investigate the method used by Lester Hill to encrypt and decrypt information using matrix multiplication and inverse matrices. Give a detailed explanation of Hill's method. Your answer needs to be supported with an example showing both the encrypting and decrypting of a message.



Task 2
Use this technique and the key matrix below to encode the message *Matrices are fun*

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 1 \\ 1 & 3 & 3 \end{bmatrix}$$

1. Find an appropriate decipher code for this encoding matrix.
2. Now write a brief message and 'send' it to another learner to decode.
3. Receive and decode a message from another learner.

Task 3
James Bond has mastered the arithmetic of matrices, so he can use coded messages. Use the deciphering matrix below to unravel the following cipher text which James has left for one of his associates.

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

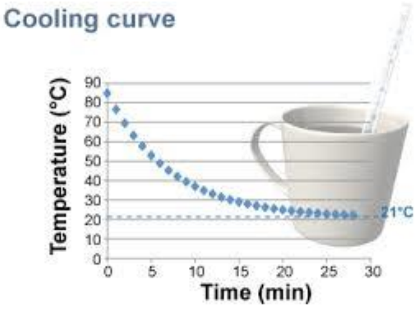
CKNMMGMWLKSOORHANXOVMJGQQDPJUO

Task 4

Research another type of coding technique, (Caesar, cypher, etc). Present a detailed description of the process, and provide a clear worked example of using it to encode and decode a message.

This application was provided by Hellyer College and used with their consent.

Newton's Law of Cooling



Ever wondered how long it would take for a can of soda to get cold enough to drink after putting it in the fridge to cool? All you need to do is to apply Newton's Law of Cooling.

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings).

Newton's Law of Cooling applies to convective heat transfer and mathematically can be represented as:

$$Rate\ of\ cooling \propto \Delta T$$

This can also be represented as the following equation:

$$\frac{dT}{dt} = -k(T - T_a), \qquad where\ T > T_a$$

Part 1 - Investigation

Determining the cooling law through observing cooling water

Materials:

- Water (heated to 100^{o}c)
- Beaker
- Thermometer

Method:

1. Record the initial temperature of the water and the ambient temperature of the room
2. Measure and record the temperature of the water every 5 (or 10) minutes
3. Graph the data and determine the nature of the equation

Results:

Conclusion:

Part 2 – Deriving the equation

Using the data collected from Part 1 determine the equation for Newton's Law of Cooling.

Part 3 – Applying your understanding

Scenario #1

The big pot of soup

As part of her job in a local restaurant, Sarah is required to make a big pot of soup late at night, just before closing time, so that there will be plenty of soup for customers the next day. She found that, whilst refrigeration was essential to preserve the soup overnight, the soup was too hot to be put directly into the industrial fridge when it was ready. The soup had just boiled at a temperature of 100oC and the regulations stated that items placed in the fridge must be n hotter than 20^{o}c.

Sarah found that by placing the pot of soup in a sink of cold water (kept running, so that its temperature was roughly a constant 5^{o}c) and stirring occasionally, she could bring the temperature of the soup to 60^{o}c in 10 minutes. How long before closing time should the soup be ready by so that Sarah could put it into the fridge and leave on time?

Points to consider

1. Look at the behaviour of the solution corresponding to the function T(t).
2. What will happen after a very long time?
3. What will the eventual temperature of the soup be?
4. How can this be understood from the expression T(t)?
5. What will happen to the term $e^{-0.054t}$?
6. Will this term be increasing or decreasing with time?
7. What will the temperature of the soup be after 1 hour?
8. Is there something analogous to a half – life in this problem?
9. What would happen if Sarah did not stir the soup occasionally?
10. Would it still cool the same way? Would Newton's Law of Cooling apply just as before?
11. Why are we assuming that the pot is well-stirred?
12. What would happen if the water in the sink was not running?
13. How would this change our assumption that the ambient temperature was a constant?

Extension

Consider investigating using cups of different thermal conductivity or changing the ambient temperature (e.g. using an ice bath).

This application was provided by Adrian Baron and used with his consent.

Markov Chain

Part 1

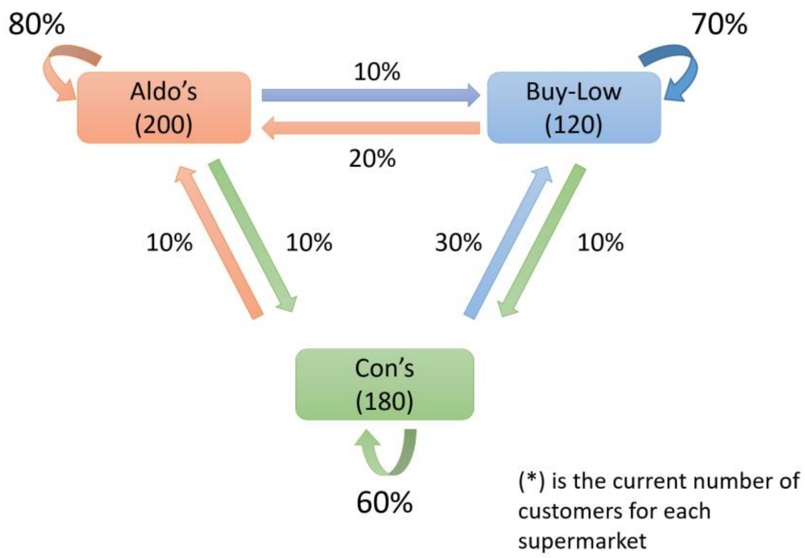
Historical background

In this part, answer the following questions as a minimum
1. Who was Andrey Markov?
2. What is a Markov chain?
3. How is the Markov Chain used in real-world situations?

Part 2

Scenario 1 – Market share of customers

Three supermarkets in a town are constantly fighting for market share. The diagram below shows the supermarkets, initial number of customers and the change in customer base each week.



$$[\text{next state}] = [\text{Matrix of transition probabilities}] \times [\text{current state}]$$

Using:

$$X_{next\ state} = PX_{current\ state}$$

where P is the probability matrix

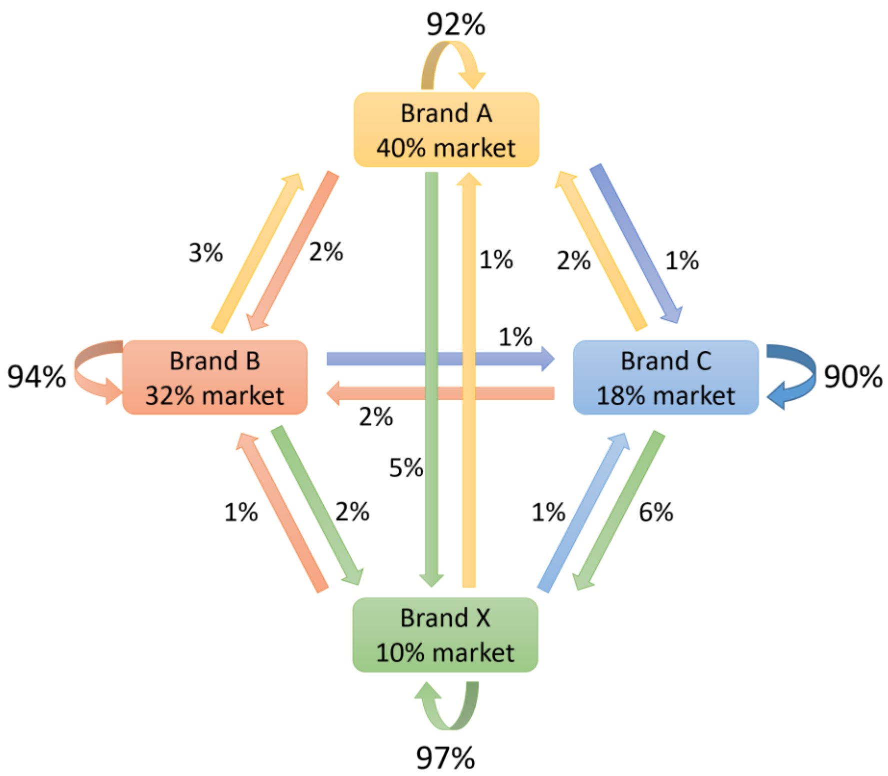
Show that:

$$X_n = P^n X_0$$

Part 3

Scenario 2 – Market Penetration

A new brand of cleaning product is introduced to the market. After much advertising the diagram below shows the probabilities for each brand over time (weekly).



Overtime, what will be the market share for each product?

Part 4

Determine (research) a situation or scenario that Markov Chain can be applied and apply Markov Chain to determine the stable distribution state.

This application was provided by Adrian Baron and used with his consent.

SUPPORTING LEARNER RESPONSES AND ELABORATIONS

There is scope in all the course units for teachers to select learning activities which will engage their learners and which will challenge them appropriately. All suggested learning strategies in this course supplement can be adapted to allow learners to develop the required knowledge and skills. Some teaching and learning strategies that are particularly relevant and effective in Mathematics Specialised include some of the following techniques and strategies.

Review prior learning

- brainstorming, individual, pair and group work
- formative assessment strategies

Introduce new material

- link topic to prior mathematical knowledge, practical applications exposure to quality visual imagery/materials through a variety of media
- investigation using a range of technologies
- application to real-world contexts

Provide demonstration, guided practice and application

- teacher demonstration, modelling and peer tutoring
- teacher scaffolding to facilitate analysis of concepts
- applications involving simulated real life and work scenarios
- use of online materials
- opportunities to develop modelling and/or problem solving skills in practical contexts

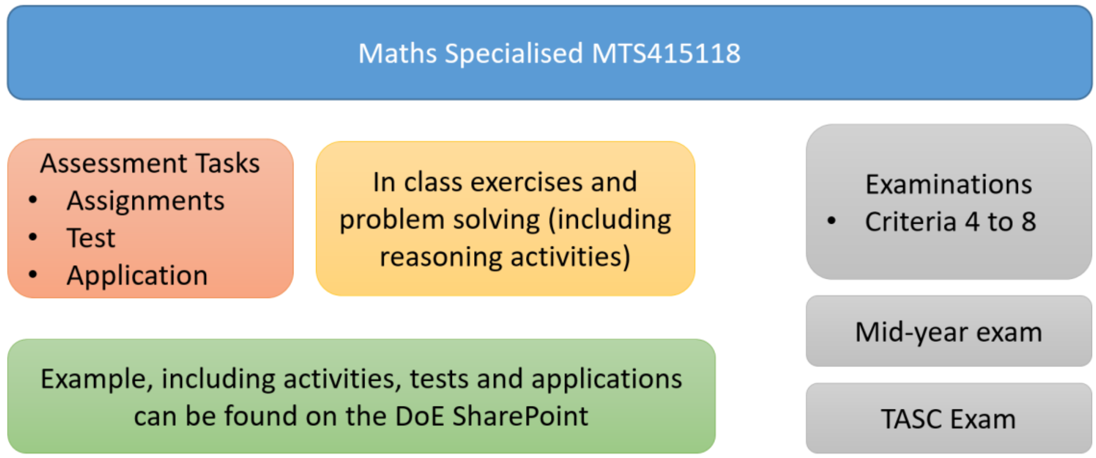
Promote independent practice and application

- research strategies and time management
- problem solving strategies
- mentoring and peer tutoring, including the use of formative assessment strategies
- practice and reinforcement of learning by ways of investigations, group activities, assessment tasks and demonstrations
- encourage responsibility for their own learning
- formative assessment strategies to provide regular and meaningful feedback
- discussions, debates and learner presentations
- applications, in the form of investigations, research and project based tasks
- opportunities to develop modelling or problem solving skills in practical contexts

Review and rehearse

- use of assignments, structured revision times for feedback and formative assessment
- assessment tasks to help build confidence and mastery of concepts and skills

WORK REQUIREMENTS



While the content criteria 4 to 8 are assessed within topics and in tests and exams, criteria 1 to 3 are internal criteria that are assessed within the context of 'in class' activities, exercises, applications and assignments. The ratings for all criteria are to be given against the standards in the curriculum document. It is expected that most learners will improve their ratings as the year progresses.

THE MINUMUM WORK REQUIREMENTS

- At least five major assessment tasks covering each of the content criteria (4 – 8)
- At least ten (10) minor assessment tasks covering each of the content criteria (4 – 8) and the general criteria (1 – 3)
- At least two (2) investigations into some applications of content areas in this course. Each is to be assessed against Criteria (1 – 3) and against (at least) one of the content Criteria (4 – 8)

RESOURCES

Recommended books

Stephen J Watson Publications (4 booklets) – Main text used by Colleges and Schools

- Linear Transformations and Matrices (updated for 2018)
- Complex Numbers
- Calculus (Differential and integral)
- Sequences and Series

Additional books

Jacaranda Publications

- Maths Quest 11 Specialist Mathematics
- Maths Quest 12 Specialist Mathematics

Alan J Sadler

- Mathematics Specialist Units 1 & 2
- Mathematics Specialist Units 3 & 4

Websites

http://nrich.maths.org/frontpage

The Mathematical Association of Victoria
https://www.mav.vic.edu.au/

Supporting Australian Mathematics Project
http://amsi.org.au/ESA_Senior_Years/seniors_years.html

Texas Instrument
https://education.ti.com/en-au/downloads-and-activities

Wofram demonstrations Project
http://demonstrations.wolfram.com/topics.html?Mathematics#2

Wolfram Math World
http://mathworld.wolfram.com/