DEPARTMENT OF EDUCATION
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Discipline-based Study

Mathematics

General Mathematics 3

COURSE DOCUMENT

DRAFT

PHASE 3 CONSULTATION

Catholic Education Tasmania

INDEPENDENT SCHOOLS TASMANIA

Years 9 to 12 Learning

Tasmanian Government

Table of Contents

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General Mathematics, 150 hours – Level 3

This course is the Level 3 component of the General Mathematics program.

Aims

The purpose of Years 9 to 12 Education is to enable all students to achieve their potential through Years 9 to 12 and beyond in further study, training or employment.

Years 9 to 12 Education enables: Personal Empowerment, Cultural Transmission, Preparation for Citizenship and Preparation for Work.

This course supports the principles of Access, Agency, Excellence, Balance, Support and Achievement as part of a range of programs that enables students to access a diverse and highly flexible range of learning opportunities suited to their level of readiness, interests and aspirations.

Courses aligned to the Years 9 to 12 Curriculum Framework belong to one of the five focus areas of Discipline-based Study, Transdisciplinary Projects, Professional Studies, Work-based Learning and Personal Futures.

General Mathematics Level 3 is a Discipline-based Study course.

Focus Area – Discipline-based Study

Discipline-based Study includes content, core concepts and big ideas; enabling deep knowledge and understanding of the content and the application of what is learned. Students consider accepted key disciplinary knowledge, apply distinctive ways of thinking and become increasingly independent learners. They use methodologies specific to the discipline to explore and strengthen their understanding of key concepts and develop deep knowledge, skills and understanding.

Discipline-based Study courses have three key features that guide teaching and learning:

- specialist knowledge
- theories and concepts and
- methodology and terminology.

In this course learners will engage with specialist knowledge, core concepts and big ideas in the strands of algebra, finance, statistics, probability and networks. Students will apply their knowledge and understanding through strategic selection and application of methodologies including problem solving, mathematical modelling and statistical investigations with and without the aid of technology.

Throughout the course, learners will demonstrate conceptual understanding through their fluency of calculation, mathematical reasoning and communication of mathematical ideas and information using appropriate conventions, terminology and representations

Rationale

The *General Mathematics* Level 3 course is designed to develop learners' understanding of concepts and techniques drawn from number including finance and algebra including sequences, networks and decision mathematics, and statistics and probability. This breadth of mathematical experience will enable learners to apply mathematical concepts and perform techniques to solve applied problems, synthesise mathematical information, and design and conduct mathematical investigations to calculate and communicate possible solutions.

The *General Mathematics* Level 3 course will enable learners the opportunity to develop the foundations for study in many disciplines at tertiary level and engage in applications of those disciplines. Mathematics and numeracy provide a way of interpreting everyday practical situations and provide the basis for many informed personal decisions.

This course will enable learners to develop their mathematical expertise such that they may contribute productively in an ever-changing global economy, with both rapid revolutions in technology and global and local social challenges. This is a key factor in ensuring Tasmania and Australia's current and emerging needs are met as an economy competing globally requires substantial numbers of professional with a strong grounding in mathematics and other disciplines of STEM. This course is designed to support learners on this pathway into tertiary education for non-STEM specific professions including teaching, social sciences, health sciences, accounting, business and marketing.

Integration of General Capabilities and Cross-Curriculum Priorities

The general capabilities addressed specifically in this course are:

- Critical and creative thinking
- Ethical understanding
- Information and communication technology capability
- Intercultural understanding
- Literacy
- Numeracy
- Personal and social capability

The cross-curriculum priorities are enabled through this course are:

- Aboriginal and Torres Strait Islander Histories and Cultures
- Asia and Australia's Engagement with Asia
- Sustainability

Course Description

In *General Mathematics* Level 3, learners will engage with concepts and techniques drawn from:

- number including finance
- algebra including sequences
- networks and decision mathematics
- statistics and probability.

Learners will use these mathematical concepts and techniques to:

- solve applied problems
- design and conduct mathematical investigations
- construct reasoned arguments.

In this course, learners will:

- recognise, model and solve real-world problems.
- develop skills and dispositions to support their engagement in further schooling, work and life
- reflect on their own learning and mathematical experiences.

This course provides a foundational pathway to study in Applications of Discrete Mathematics 4 and tertiary education in non-STEM specific fields.

Pathways

The *General Mathematics* Level 3 course enables learning continuity from Year 10 Australian Curriculum: Mathematics for learners who have achieved a 'B' rating or higher. Additionally, learners who have successfully undertaken the currently accredited TASC course *General Mathematics* – Foundation MTG215114 or the Level 2 component of the General Mathematics suite of courses under development could progress into Level 3 of this course. Finally, students who have successfully completed Level 3 of the *Essential Mathematics* course under development may progress into this course if recommended through ongoing course counselling at their provider of education.

General Mathematics Level 3 will provide a pathway into additional mathematics study of the TASC course *Applications of Discrete Mathematics* (under development) for learners wishing to apply mathematics knowledge in a Transdisciplinary Project course and provides foundational knowledge for students wishing to pursue tertiary education in non-STEM specific fields.

Course Requirements

Resources

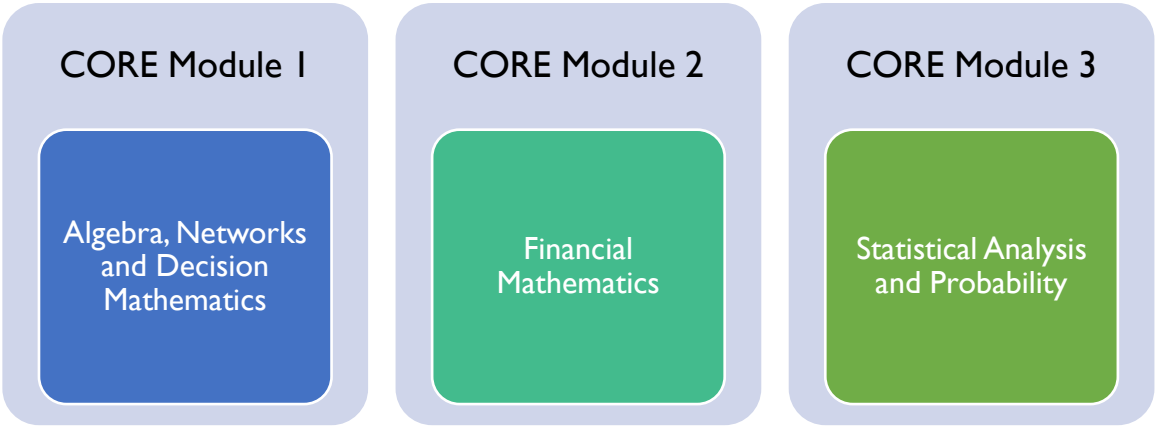
- Students will require access to graphics calculators

Course Structure, Delivery and Progression

Structure

This course consists of three 50-hour modules.

Modules available
Core Module 1: Algebra, Networks and Decision Mathematics
Core Module 2: Financial Mathematics
Core Module 3: Statistical Analysis and Probability



Delivery

There is no specific recommended delivery sequence for the modules.

Developmental Progression

At both the module and course level the learner is introduced to and builds upon key ideas, concepts, skills, knowledge and understanding leading to performance of understanding reflected in the work requirements.

Individual modules have a developmental progression that introduces, builds upon and culminates in a performance of understanding in the work requirements. Between modules there is also a developmental progression that leads to a culminating performance of understanding in the final work requirements.

Module 1 - Algebra, Networks and Decision Mathematics

This Module contains two topics:

- Growth and decay in sequences
- Graphs, networks and decision mathematics.

'Growth and decay in sequences' employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population, or the decrease in the value of a car over time.

'Graphs, networks and decision mathematics' involves the graphical representation and modelling of situations as an approach to decision-making processes. Knowledge of networks enables development of a logical sequence of tasks or a clear understanding of connections between people or items and project planning and management tools such as critical path analysis and the 'maximum-flow, minimum-cut' theorem. Study of this topic is important in developing students' ability to interpret a set of connections or sequence of tasks as a concise diagram in order to solve related problems and to use critical path analysis in the optimisation of real-life problems.

Module 1 Learning Outcomes

On successful completion of this module, learners will be able to:

1. define and explain key knowledge and concepts and apply a range of related mathematical techniques and procedures to solve practical problems from a range of everyday and real-life contexts
2. select and apply mathematical processes to open-ended practical contexts, analyse and discuss the obtained results
3. apply mathematical reasoning to develop logical arguments, explain and justify the reasonableness of solutions
4. interpret mathematical objects and information in a variety of contexts and evaluate the effectiveness of its use.

Module 1 Content

Growth and decay in sequences

Subtopics:

- the arithmetic sequence
- the geometric sequence
- sequences generated by first-order linear recurrence relations.

The arithmetic sequence:

Key knowledge and skills:

- use recursion to generate an arithmetic sequence (ACMGM067)
- display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations (ACMGM068)
- deduce a rule for the nth term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions (ACMGM069)
- use arithmetic sequences to model and analyse practical situations involving linear growth or decay; for example, analysing a simple interest loan or investment, calculating a taxi fare based on the flag fall and the charge per kilometre, or calculating the value of an office photocopier at the end of each year using the straight-line method or the unit cost method of depreciation. (ACMGM070).

The geometric sequence:

Key knowledge and skills:

- use recursion to generate a geometric sequence (ACMGM071)
- display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations (ACMGM072)
- deduce a rule for the nth term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions (ACMGM073)
- use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay; for example, analysing a compound interest loan or investment, the growth of a bacterial population that doubles in size each hour, the decreasing height of the bounce of a ball at each bounce; or calculating the value of office furniture at the end of each year using the declining (reducing) balance method to depreciate. (ACMGM074).

Sequences generated by first-order linear recurrence relations:

Key knowledge and skills:

- use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form (ACMGM075)
- recognise that a sequence generated by a first-order linear recurrence relation can have a long-term increasing, decreasing or steady-state solution (ACMGM076)
- use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems involving growth or decay; for example, investigating the growth of a trout population in a lake recorded at the end of each year and where limited recreational fishing is permitted.

Graphs, networks and decision mathematics

Subtopics:

- graphs
- paths and cycles
- trees and minimum connector problems
- critical path analysis
- flow networks and assignment problems.

Graphs:

Key knowledge and skills:

- explain the meanings of the terms: graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph), arc, weighted graph, and network (ACMGM078)
- identify practical situations that can be represented by a network, and construct such networks; for example, trails connecting camp sites in a National Park, a social network, a transport network with one-way streets, a food web, the results of a round-robin sporting competition (ACMGM079)

- construct an adjacency matrix from a given graph or digraph. (ACMGM080)
- explain the meaning of the terms: planar graph, and face (ACMGM081)
- apply Euler's formula, $V + F - E = 2$, to solve problems relating to planar graphs. (ACMGM082).

Paths and cycles:

Key knowledge and skills:

- explain the meaning of the terms: walk, trail, path, closed walk, closed trail, cycle, connected graph, and bridge (ACMGM083)
- investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only) (ACMGM084)
- explain the meaning of the terms Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems; for example, the Königsberg Bridge problem, planning a garbage bin collection route (ACMGM085)
- explain the meaning of the terms Hamiltonian graph and semi-Hamiltonian graph and use these concepts to investigate and solve practical problems; for example, planning a sight-seeing tourist route around a city, the travelling-salesman problem (by trial-and-error methods only). (ACMGM086)

Trees and minimum connector problems:

Key knowledge and skills:

- explain the meaning of the terms tree and spanning tree identify practical examples (ACMGM101)
- identify a minimum spanning tree in a weighted connected graph either by inspection or by using Prim's algorithm (ACMGM102)
- use minimal spanning trees to solve minimal connector problems; for example, minimising the length of cable needed to provide power from a single power station to substations in several towns. (ACMGM103).

Critical path analysis:

Key knowledge and skills:

- construct a network to represent the durations and interdependencies of activities that must be completed during the project; for example, preparing a meal (ACMGM104)
- use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project (ACMGM105)
- use ESTs and LSTs to locate the critical path(s) for the project (ACMGM106)
- use the critical path to determine the minimum time for a project to be completed (ACMGM107)
- calculate float times for non-critical activities. (ACMGM108).

Flow networks and assignment problems:

Key knowledge and skills:

- solve small-scale network flow problems including the use of the 'maximum-flow minimum- cut' theorem; for example, determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank (the source) to a terminal (the sink). (ACMGM109)
- use a bipartite graph and/or its tabular or matrix form to represent an assignment/ allocation problem; for example, assigning four swimmers to the four places in a medley relay team to maximise the team's chances of winning (ACMGM110)
- determine the optimum assignment(s), by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems. (ACMGM111).

Module 1 Work Requirements

The work requirements of a course are processes, products or performances that provide a significant demonstration of achievement that is measurable against the course's standards. Work requirements need not be the sole form of assessment for a module.

This module includes an extended application of knowledge through a problem solving and/or mathematical modelling task as the work requirement.

See Appendix 3 for summary of Work Requirement specifications for this course.

Module 1 Assessment

This module will assess criteria 1, 2, 3, 4.

Module 2 - Financial Mathematics

This module contains two topics:

- rates and ratio
- investment, loans and annuities.

'Rates and ratio' provides students with the mathematical skills and understanding to solve problems relating to calculations, representation and comparison of financial performance ratios and exchange rates. These calculations find application in a wide range of practical situations including travel, business and enterprise.

'Investment, loans and annuities' aims to provide students with sufficient knowledge and understanding of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage, the depreciation on plant equipment, contributions to superannuation and making investments.

Study of this module will assist students to develop awareness of mechanisms to optimise their financial position, both now and into the future, justifying their thinking and reasoning mathematically.

Module 2 Learning Outcomes

On successful completion of this module, learners will be able to:

1. define and explain key knowledge and concepts and apply a range of related mathematical techniques and procedures to solve practical problems from a range of everyday and real-life contexts
2. select and apply mathematical processes to open-ended practical contexts, analyse and discuss the obtained results
3. apply mathematical reasoning to develop logical arguments, explain and justify the reasonableness of solutions
5. manage self, take responsibility for their own learning and evaluate their mathematical development.

Module 2 Content

Rates and ratio:

Key knowledge and skills

- solve practical finance problems involving ratio, for example:
  - unit costing as a pricing method
  - financial performance ratios such as current ratio, debt ratio, profit margin ratio and return on equity
- solve practical finance problems involving exchange rates, for example:
  - converting between AUD and foreign currencies, including calculating amount lost on bank charges (buy and sell rates)
  - calculating the financial impact on the import and /or export of goods between countries as the exchange rate fluctuates

Investment, loans and annuities.

Subtopics:

- compound interest investments and loans
- reducing balance loans
- annuities and perpetuities

Compound interest investments and loans:

Key knowledge and skills:

- use a recurrence relation to model a compound interest loan or investment, and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment (ACMGM094)
- calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)
- solve, with and without the aid of a calculator or computer-based financial software, problems involving compound interest loans or investments, for example:
  - calculate the future value (FV) or present value (PV) and the interest rate (r) of a compound interest investment using the formula $FV = PV(1 + r)^n$
  - determine the number of compounding periods for an investment to exceed a given value

    - investigate the effect of varying the interest rate, the term or the compounding period on the future value of an investment, using technology
    - compare and contrast different investment strategies, performing appropriate calculations when needed
- solve practical problems involving compounding, for example determine the impact of inflation on prices and wages
- work with shares and calculate the appreciated value of items, for example antiques
    - record and graph the price of a share over time
    - calculate the dividend paid on a portfolio of shares, and the dividend yield (excluding franked dividends)

Reducing balance loans:

Key knowledge and skills:

- use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan (ACMGM097)
- calculate the depreciation of an asset using the declining-balance method using the formula $S = V_0(1-r)^n$, where S is the salvage value of the asset after n periods, V_0 is the initial value of the asset, r is the depreciation rate per period, expressed as a decimal, and n is the number of periods, as an application of the compound interest formula
- with and without the aid of a financial calculator or computer-based financial software, solve practical problems involving reducing balance loans, for example determining the total loan amount and monthly repayments
- recognise credit cards as an example of a reducing balance loan and solve practical problems relating to credit cards
    - identify the various fees and charges associated with credit card usage
    - compare credit card interest rates with interest rates for other loan types
    - interpret credit card statements, recognising the implications of only making the minimum payment
    - understand what is meant by an interest-free period
    - calculate the compounding interest charged on a retail purchase, transaction or the outstanding balance for a given number of days, using technology or otherwise

Annuities and perpetuities:

Key knowledge and skills:

- identify an annuity as an investment account with regular, equal contributions and interest compounding at the end of each period, or as a single sum investment from which regular, equal withdrawals are made
- with the aid of a financial calculator or computer-based financial software, model an annuity as a recurrence relation, and investigate (numerically or graphically) the effect of varying the amount and frequency of each contribution, the interest rate or the payment amount on the duration and/or future value of the annuity
- use a table of interest factors to perform annuity calculations, e.g. calculating the present or future value of an annuity, the contribution amount required to achieve a given future value or the single sum that would produce the same future value as a given annuity

Module 2 Work Requirements

The work requirements of a course are processes, products or performances that provide a significant demonstration of achievement that is measurable against the course's standards. Work requirements need not be the sole form of assessment for a module.

This module includes an extended application of knowledge through a problem solving and/or mathematical modelling task as the work requirement.

See Appendix 3 for summary of Work Requirement specifications for this course.

Module 2 Assessment

This module will assess criteria 1, 2, 3, 5.

Module 3 - Statistical Analysis and Probability

This module contains two topics:

- statistical analysis
- counting and probability.

'Statistical analysis' provides students with the opportunity to follow the statistical investigation process to identify, analyse and describe associations between pairs of variables, including using the least-squares method as a tool for modelling and analysing linear associations. Additionally, this topic introduces students to the concepts and techniques of time series analysis.

'Counting and probability' provides students with knowledge and understanding of the fundamentals of probability and the terminology related to sets and events, allowing them to solve practical problems involving combinations, conditional probability and independence.

Study of this module will assist students to develop awareness of statistical relationships and to recognise and model situations involving chance. They will be required to demonstrate skills in constructing logical argument based on mathematical reasoning.

Module 3 Learning Outcomes

On successful completion of this module, learners will be able to:

1. define and explain key knowledge and concepts and apply a range of related mathematical techniques and procedures to solve practical problems from a range of everyday and real-life contexts
2. select and apply mathematical processes to open-ended practical contexts, analyse and discuss the obtained results
3. apply mathematical reasoning to develop logical arguments, explain and justify the reasonableness of solutions
6. communicate and represent mathematical information and apply mathematical conventions.

Statistical analysis

Subtopics:

- bivariate data analysis
- time series analysis.

Bivariate data analysis:

Key knowledge and skills:

- review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results. (ACMGM048)
- construct two-way frequency tables and determine the associated row and column sums and percentages (ACMGM049)
- use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association (ACMGM050)
- describe an association in terms of differences observed in percentages across categories in a systematic and concise manner and interpret this in the context of the data. (ACMGM051)
- construct a scatterplot to identify patterns in the data suggesting the presence of an association (ACMGM052)
- describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak) (ACMGM053)
- calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association. (ACMGM054)
- identify the response variable and the explanatory variable (ACMGM055)
- use a scatterplot to identify the nature of the relationship between variables (ACMGM056)
- model a linear relationship by fitting a least-squares line to the data (ACMGM057)
- use a residual plot to assess the appropriateness of fitting a linear model to the data (ACMGM058)
- interpret the intercept and slope of the fitted line (ACMGM059)
- use the coefficient of determination to assess the strength of a linear association in terms of the explained variation (ACMGM060)
- use the equation of a fitted line to make predictions (ACMGM061)
- distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation (ACMGM062)
- write up the results of the above analysis in a systematic and concise manner. (ACMGM063)
- recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them (ACMGM064)
- identify possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable, and communicate these explanations in a systematic and concise manner. (ACMGM065)
- implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or between two numerical variables; for example, is there an association between attitude to capital punishment

(agree with, no opinion, disagree with) and sex (male, female)? is there an association between height and foot length? (ACMGM066).

Time series analysis:

Key knowledge and skills:

- construct time series plots (ACMGM087)
- describe time series plots by identifying features such as trend (long term direction), seasonality (systematic, calendar-related movements), and irregular fluctuations (unsystematic, short term fluctuations), and recognise when there are outliers, for example, one-off unanticipated events. (ACMGM088)
- smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process (ACMGM089)
- calculate seasonal indices by using the average percentage method (ACMGM090)
- deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process (ACMGM091)
- fit a least-squares line to model long-term trends in time series data. (ACMGM092)
- implement the statistical investigation process to answer questions that involve the analysis of time series data. (ACMGM093).

Counting and probability

Subtopics:

- combinations
- language of events and sets
- fundamentals of probability
- conditional probability.

Combinations:

Key knowledge and skills:

- understand the notion of a combination as an unordered set of r objects taken from a set of n distinct objects (ACMMM044)
- use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects (ACMMM045)
- expand $(x + y)^n$ for small positive integers n (ACMMM046)
- recognise the numbers $\binom{n}{r}$ as binomial coefficients, that is, as coefficients in the expansion of $(x + y)^n$ (ACMMM047)
- use Pascal's triangle and its properties. (ACMMM048).

Language of events and sets:

Key knowledge and skills:

- review the concepts and language of outcomes, sample spaces and events as sets of outcomes (ACMMM049)
- use set language and notation for events, including $\bar{A}$ (or A') for the complement of an event A, $A \cap B$ for the intersection of events A and B, and $A \cup B$ for the union, and recognise mutually exclusive events (ACMMM050)

- use everyday occurrences to illustrate set descriptions and representations of events and set operations. (ACMMM051).

Fundamentals of probability:

Key knowledge and skills:

- review probability as a measure of 'the likelihood of occurrence' of an event (ACMMM052)
- review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty (ACMMM053)
- review the rules: $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054)
- use relative frequencies obtained from data as point estimates of probabilities. (ACMMM055)

Conditional probability and independence:

Key knowledge and skills:

- understand the notion of a conditional probability and recognise and use language that indicates conditionality (ACMMM056)
- use the notation $P(A|B)$ and the formula $P(A|B) = P(A \cap B)/P(B)$ (ACMMM057)
- understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$ (ACMMM058)
- establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence (ACMMM059)
- use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events. (ACMMM060).

Module 3 Work Requirements

The work requirements of a course are processes, products or performances that provide a significant demonstration of achievement that is measurable against the course's standards. Work requirements need not be the sole form of assessment for a module.

This module includes X short response(s) / extended Response(s) /project / investigation / inquiry / This module includes an extended application of knowledge through a statistical investigation as the work requirement.

See Appendix 3 for summary of Work Requirement specifications for this course.

Module 3 Assessment

This module will assess criteria 1, 2, 3, 6.

Assessment

Criterion-based assessment is a form of outcomes assessment that identifies the extent of learner achievement at an appropriate end-point of study. Although assessment – as part of the learning program – is continuous, much of it is formative, and is done to help learners identify what they need to do to attain the maximum benefit from their study of the course. Therefore, assessment for summative reporting to TASC will focus on what both teacher and learner understand to reflect end-point achievement.

The standard of achievement each learner attains on each criterion is recorded as a rating 'A', 'B', or 'C', according to the outcomes specified in the standards section of the course.

A 't' notation must be used where a learner demonstrates any achievement against a criterion less than the standard specified for the 'C' rating.

A 'z' notation is to be used where a learner provides no evidence of achievement at all.

Internal assessment of all criteria will be made by the provider. Providers will report the learner's rating for each criterion to TASC.

TASC will supervise the external assessment of designated criteria which will be indicated by an asterisk (*). The ratings obtained from the external assessments will be used in addition to internal ratings from the provider to determine the final award.

Criteria

	Module 1	Module 2	Module 3	Notes
Criteria assessed	1,2,3,4	1,2,3,6	1,2,3,5	Three common in all modules and one focus criterion per module.

The assessment for *General Mathematics* Level 3 will be based on the degree to which the learner can:

1. *define and explain key knowledge and concepts and apply a range of related mathematical techniques and procedures to solve practical problems from a range of everyday and real-life contexts
2. *select and apply mathematical processes to open-ended practical contexts, analyse and discuss the obtained results
3. *apply mathematical reasoning to develop logical arguments, explain and justify the reasonableness of solutions
4. communicate and represent mathematical information and apply mathematical conventions
5. interpret mathematical objects and information in a variety of contexts and evaluate the effectiveness of its use
6. manage self, take responsibility for their own learning and evaluate their mathematical development.

*denotes criteria that are both internally and externally assessed.

Standards

Criterion 1*: define and explain key knowledge and concepts and apply a range of related mathematical techniques and procedures to solve practical problems

This criterion is both internally and externally assessed.

Rating C	Rating B	Rating A
selects, recalls and uses some mathematical facts, rules and definitions to describe mathematical situations	consistently selects, recalls and uses facts, rules, definitions and procedures correctly to describe mathematical situations	consistently selects, recalls and uses facts, rules, definitions and procedures correctly to describe and explain mathematical situations
comprehends and applies aspects of mathematical concepts and techniques to solve some problems in simple familiar situations	comprehends and applies mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations	comprehends and applies mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations
applies mathematical and statistical models to routine problems	selects and applies mathematical and statistical models to routine and non-routine problems	develops, selects and applies mathematical and statistical models to routine and non-routine problems in a variety of contexts
uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems.	uses digital technologies appropriately to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems.	uses digital technologies effectively to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems in a variety of contexts.

Criterion 2*: select and apply mathematical processes to open-ended practical contexts^, analyse, discuss and draw conclusions from the results obtained

This criterion is both internally and externally assessed.

Rating C	Rating B	Rating A
recognises how mathematics is used in everyday situations, making some connections between mathematics and open-ended practical contexts	identifies and describes how mathematics is used in everyday situations and formulates mathematical approaches to explore open-ended practical contexts	describes and explains the mathematics embedded in everyday situations and formulates mathematical approaches to explore open-ended practical contexts
uses mathematical applications and processes to find solutions or results to open-ended practical problems or investigations	selects and applies a range of mathematical applications and processes to find accurate solutions or results to open-ended practical problems or investigations	strategically selects and applies a broad range of mathematical applications and processes to find efficient and accurate solutions or results to open-ended practical problems or investigations
represents and analyses results that provide some clarity and drawn conclusions that may be plausible but lack detailed supporting evidence	uses valid mathematical representations, provides detailed analysis of results and draws plausible conclusions with supporting evidence that are appropriate to the context	strategically selects appropriate mathematical representations, provides rich and accurate analysis of results and draws valid evidence-based conclusions that are appropriate to the context
describes how the mathematical applications and processes used and the outcomes obtained were suitable for the context	selects and justifies use of mathematical applications and processes and identifies how the outcomes obtained may impact the dignity and wellbeing of individuals and communities.	critically reflects and evaluates the impact of selected processes and the outcomes obtained upon the dignity and wellbeing of individual or communities.

^open-ended practical contexts require problem-solving, modelling or investigative techniques or approaches.

Criterion 3*: apply mathematical reasoning to develop logical arguments, explain and justify the reasonableness of solutions

This criterion is both internally and externally assessed.

Rating C	Rating B	Rating A
identifies problem elements and makes inferences that may be able to be tested mathematically	identifies and explains problem elements to make informed inferences that can be tested mathematically	explores and links problem elements to make logical inferences that can be tested mathematically
forms arguments based on some mathematical ideas to support or refute claims and identifies possible responses	develops logical mathematical arguments to support or refute conclusions and outlines a chosen response	develops logical mathematical arguments to evaluate conclusions and justify a response
describes the reasonableness of the results and solutions to routine problems	explains the reasonableness of the results and solutions to routine and non-routine problems	evaluates and explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts
identifies limitations of models used when developing solutions to routine problems.	identifies and explains limitations of models used when developing solutions to routine problems.	identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems.

Criterion 4: communicate and represent mathematical information and apply mathematical conventions

Rating C	Rating B	Rating A
communicates mathematical and statistical arguments using appropriate language	communicates reasoned mathematical and statistical judgments and arguments using appropriate language	communicates reasoned mathematical and statistical judgments and arguments using appropriate and concise language
uses mathematical conventions, systems and constructs based on definitions and rules when prompted	uses mathematical conventions, systems and constructs including manipulation and use of symbolic expressions and rules appropriately on most occasions	uses mathematical conventions, systems and constructs including manipulation and use of symbolic expressions, rules and formal systems accurately and purposefully
represents and interprets mathematical and statistical information in numerical, graphical and symbolic form in routine problems with and without technology	represents and interprets mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems with and without technology	represents and interprets mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts with and without technology
considers how culture, context and experience may impact perception when expressing ideas.	considers culture, context and experience and provides multiple perspectives when expressing ideas.	considers culture, context and experience, empathises with others and develops multiple perspectives when expressing ideas.

Criterion 5: interpret mathematical objects and information in a variety of contexts and evaluate the effectiveness of its use

Rating C	Rating B	Rating A
explores routine familiar practical problems or scenarios and frames them in mathematical terms	interprets routine familiar and complex familiar practical problems or scenarios and frames them in mathematical terms	interprets complex familiar and non-familiar practical problems or scenarios and frames them in mathematical terms
identifies situations where mathematical information has been used to support decision-making within cultural, environmental, political or economic systems	analyses various ways that mathematical information has been used to support decision-making within cultural, environmental, political or economic systems	evaluates how mathematical information has been used to support decision-making within cultural, environmental, political or economic systems
accesses, manages and acknowledges information from digital and non-digital sources to develop mathematical ideas	accesses, synthesises and appropriately acknowledges information taken from a variety of digital and non-digital sources to develop mathematical ideas	evaluates authenticity, reliability and validity of information taken from a variety of digital and non-digital sources to develop mathematical ideas
identifies and describes how the use of technology can affect outcomes obtained in routine contexts.	identifies and discusses the inputs and outputs of technology and describes how the use of technology can affect outcomes obtained in simple non-routine contexts.	interprets and evaluates the inputs and outputs of technology, including critically reflecting on and evaluating the technology used and the outcomes obtained relative to personal, contextual and real-world implications.

Criterion 6: manage self, take responsibility for their own learning and evaluate their mathematical development

Rating C	Rating B	Rating A
recognises own learning strengths and weaknesses and establishes processes to plan, monitor and assess one's understanding and performance	analyses own learning strengths and weaknesses in order to establish processes used to plan, monitor and assess one's understanding and performance	critically reflects upon own learning strengths and weaknesses in order to establish processes used to plan, monitor and assess one's understanding and performance
sets goals and timelines and monitors with support	monitors and analyses progress towards meeting goals and timelines	monitors and evaluates progress towards meeting goals and timelines, and plans future actions
shows some ability to organise and plan in order to manage resources and complete set tasks	displays organisational, planning and self-management skills to manage resources and consistently complete tasks	selects and displays effective organisational, planning and self-management skills to manage resources and complete all learning tasks
is generally positive and with support will persevere through challenges.	demonstrates a positive disposition, can adapt to new situations and generally displays perseverance and resilience and seeks help as required.	demonstrates optimism, flexibility and resilience when adapting to new situations and transitions including persevering through challenges and seeking help appropriately as required.

Quality Assurance

- This will be determined by TASC at time of accreditation.

Qualifications and Award Requirements

The final award will be determined by the Office of Tasmanian Assessment, Standards and Certification from the 9 ratings (6 ratings from the internal assessment and 3 ratings from the external assessment).

The minimum requirements for an award in *General Mathematics* Level 3 are as follows:

EXCEPTIONAL ACHIEVEMENT (EA)
8'A' ratings, 1 'B' rating (2 'A' ratings, 1 'B' rating from external assessment)

HIGH ACHIEVEMENT (HA)
4 'A' ratings, 4 'B' ratings, 1 'C' ratings (1 'A' rating, 1 'B' rating and 1 'C' rating from external assessment)

COMMENDABLE ACHIEVEMENT (CA)
4 'B' ratings, 4 'C' ratings (1 'B' ratings, 2 'C' ratings from external assessment)

SATISFACTORY ACHIEVEMENT (SA)
7 'C' ratings (2 'C' ratings from external assessment)

PRELIMINARY ACHIEVEMENT (PA)
5 'C' ratings

A learner who otherwise achieves the ratings for a CA (Commendable Achievement) or SA (Satisfactory Achievement) award but who fails to show any evidence of achievement in one or more criteria ('z' notation) will be issued with a PA (Preliminary Achievement) award.

Course Evaluation

- This will be confirmed by time of accreditation.

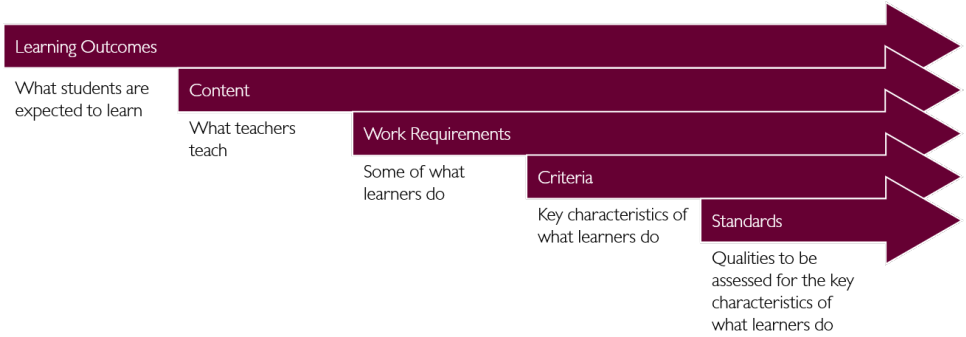
Course Developer

This course has been developed by the Department of Education's Years 9 to 12 Learning Unit in collaboration with Catholic Education Tasmania and Independent Schools Tasmania.

Accreditation and Version History

- Details to be determined by TASC at time of accreditation.

Appendix 1 - Line of Sight



Learning Outcomes	Course Content	Work Requirements	Criteria	Standards	General Capabilities (GC)
1. Define and explain key knowledge and concepts and apply a range of related mathematical techniques and procedures to solve practical problems from a range of everyday and real-life contexts.	Module 1, 2, 3	Module 1, 2, 3	C 1	E 1, 2, 3, 4	GC:
2. Select and apply mathematical processes to open-ended practical contexts, analyse and discuss the obtained results.	Module 1, 2, 3	Module 1, 2, 3	C 2	E 1, 2, 3, 4	GC:
3. Apply mathematical reasoning to develop logical arguments, explain and justify the reasonableness of solutions.	Module 1, 2, 3	Module 1, 2, 3	C 3	E 1, 2, 3, 4	GC:
4. Communicate and represent mathematical information and apply mathematical conventions.	Module 1	Module 1	C 4	E 1, 2, 3, 4	GC:

5. Interpret mathematical objects and information in a variety of contexts and evaluate the effectiveness of its use.	Module 2	Module 2	C 5	E 1, 2, 3, 4	GC: 
6. Manage self, take responsibility for their own learning and evaluate their mathematical development.	Module 3	Module 3	C 6	E 1, 2, 3, 4	GC: 

Appendix 2 - Alignment to Curriculum Frameworks

Links to Foundation to Year 10:

The General Mathematics suite provides students with a breadth of mathematical and statistical experience that encompasses and builds on all three strands of the F-10 curriculum.

Alignment to Australian Curriculum Senior Secondary Framework:

A vast majority of content in this course is drawn from Unit 3 and Unit 4 of the Australian Curriculum Framework: General Mathematics.

Unit 3 has three topics: 'Bivariate data analysis', 'Growth and decay in sequences', and 'Graphs and networks'. 'Bivariate data analysis' introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including using the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process. 'Growth and decay in sequences' employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4. 'Graphs and networks' introduces students to the language of graphs and the way in which graphs, represented as a collection of points and interconnecting lines, can be used to analyse everyday situations such as a rail or social network.

Unit 4 has three topics: 'Time series analysis', 'Loans, investments and annuities', and 'Networks and decision mathematics'. 'Time series analysis' continues students' study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process. 'Loans and investments' aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments. 'Networks and decision mathematics' uses networks to model and aid decision making in practical situations.

Additionally, the topic 'Counting and probability' is aligned to the content described in the Australian Curriculum Framework Mathematical Methods Unit 1, Topic 3. This enables study of probability and statistics with a review of the fundamentals of probability, and the introduction of the concepts of conditional probability and independence.

Appendix 3 - Work Requirements

Combined Modules 1 and 2 Work Requirements Specifications

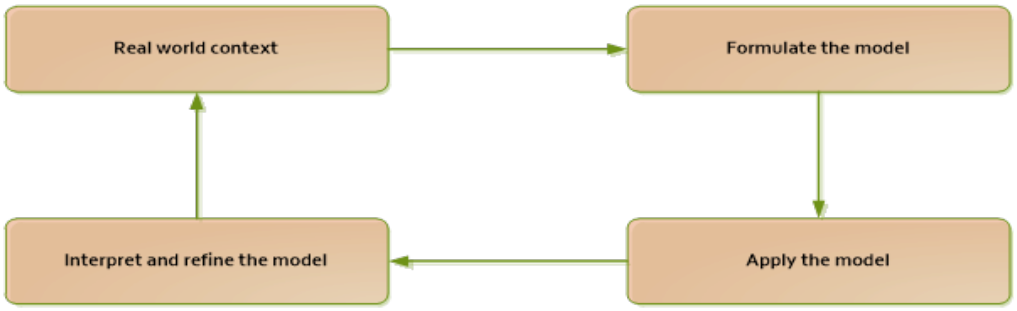
The work requirements for modules 1 and 2 require learners to employ mathematical modelling and/or problem-solving processes to investigate open-ended contexts.

These processes are defined as follows:

Mathematical modelling

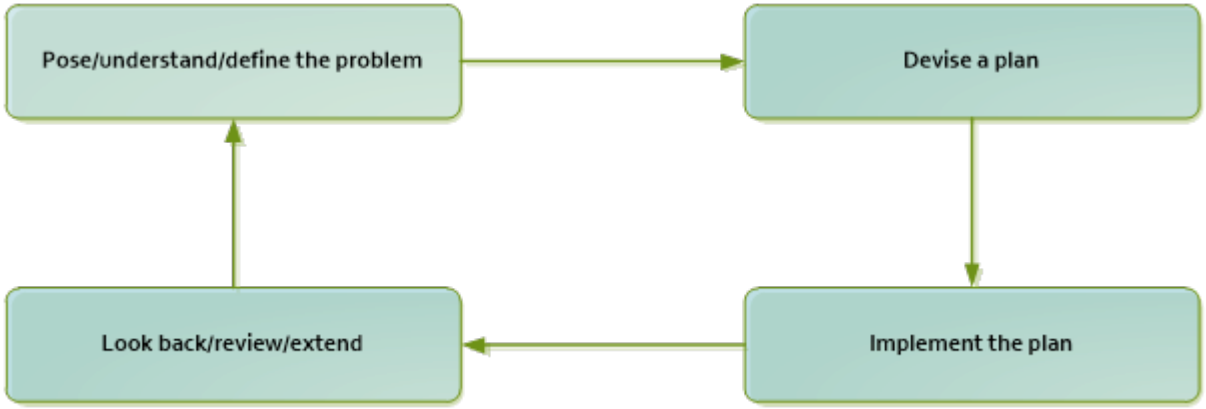
- Mathematical modelling is the process of using mathematical constructs, structures and techniques to represent and describe a real-world context or system, in a simple and concise way that enables one to investigate features and characteristics of its behaviour, analyse particular aspects or solve problems of interest, and to make predictions related to the context or system.

- A simple diagrammatic representation of the mathematical modelling process is shown below.



Problem-solving:

- Problem solving is a process that occurs in a context where a question, task or issue needs to be solved or resolved, and there is a motivation, but not yet the means, to do so.
- Questions or tasks for which there are already recognised methods or approaches for solution or resolution, do not require problem-solving in this sense.
- In mathematics problems are generated from questions, conjectures and hypotheses within and across areas of study. New problems may arise in their own right, or as a variation, re-formulation, extension or generalisation of a known problem or class of problems.
- A simple diagrammatic representation of the problem-solving process, adapted from *How to Solve It* (Polya, 1945, Princeton University Press) follows.



- Mathematical modelling and problem-solving are complementary processes. Developing a model may be a strategy that is employed to solve a problem, and problem-solving may be required in developing and applying aspects of a model.

Module 1 Work Requirements Specifications

Focus Area: Discipline-based study
Title of Work Requirement: Mathematical modelling and/or problem-solving task 1
Mode /Format: External response
Learning Outcomes: 1,2,3,5
Description: Learners will engage in problem solving and/or mathematical modelling of a real-world context involving growth and decay in sequences, graphs and networks or critical path analysis. In preparation and alongside this task it is likely that shorter practical activities will be engaged. These are designed to support the depth of understanding and engagement in the extended response.
Size: Maximum of 1500 words (and supporting mathematical calculations) – 8 to 10 hours of class time including support tasks.

Timing: No specified timing
External agencies: Involvement at teacher discretion
Relevant Criterion/criteria:

- Criterion 1: elements 1, 2, 3 and where relevant 4
- Criterion 2: all standard elements
- Criterion 3: all standard elements
- Criterion 5: all standard elements

Module 2 Work Requirements Specifications

Focus Area: Discipline-based study
Title of Work Requirement: Mathematical modelling and/or problem-solving task 2
Mode /Format: Extended response
Learning Outcomes: 1,2,3,6
Description: Learners will engage in problem solving and/or mathematical modelling of a real-world context involving growth and decay in sequences, graphs and networks or critical path analysis. In preparation and alongside this task it is likely that shorter practical activities will be engaged. These are designed to support the depth of understanding and engagement in the extended response.
Size: Maximum of 1500 words (and supporting mathematical calculations) – 8 to 10 hours of class time including support tasks.
Timing: No specified timing
External agencies: Involvement at teacher discretion
Relevant Criterion/criteria:

- Criterion 1: elements 1, 2, 3 and where relevant 4
- Criterion 2: all standard elements
- Criterion 3: all standard elements
- Criterion 6: all standard elements

Module 3 Work Requirements Specifications

Focus Area: Discipline-based study
Title of Work Requirement: Statistical investigation
Mode /Format: Investigation
Learning Outcomes: 1,2,3,4
Description: Learners will engage in a statistical investigation of a given data set with several variables. The task has three components of increasing complexity:
the construction, description and interpretation of data plots, including smoothed plots where time series data is used
the calculation and interpretation of summary statistics, including seasonal indices and their application where time series data is used
the modelling of linear associations, or trends where time series data is used, including the use of data transformation as appropriate.
Size: Maximum of 2000 words (and supporting mathematical calculations) or 1000 word and 10-minute multimodal presentation – 10 -12 hours of class time.
Timing: No specified timing
External agencies: Involvement at teacher discretion
Relevant Criterion/criteria:

- Criterion 1: all standard elements
- Criterion 2: all standard elements

- Criterion 3: all standard elements
- Criterion 4: all standard elements

Relationship to External Assessment: External assessment will include a folio of students best illustrative practices of Criteria 2 and 3 taken from any of the three work requirements or other sources.

Appendix 4 – General Capabilities and Cross-Curriculum Priorities

Learning across the curriculum content, including the cross-curriculum priorities and general capabilities, assists students to achieve the broad learning outcomes defined in the *Alice Springs (Mparntwe) Education Declaration (December 2019)*.

General Capabilities:

The general capabilities play a significant role in the Australian Curriculum in equipping young Australians to live and work successfully in the twenty-first century.

In the Australian Curriculum, capability encompasses knowledge, skills, behaviours and dispositions. Students develop capability when they apply knowledge and skills confidently, effectively and appropriately in complex and changing circumstances, in their learning at school and in their lives outside school.

The general capabilities include:

- Critical and creative thinking
- Ethical understanding
- Information and communication technology capability
- Intercultural understanding
- Literacy
- Numeracy
- Personal and social capability

Cross-Curriculum Priorities:

Cross-curriculum priorities enable students to develop understanding about and address the contemporary issues they face, for their own benefit and for the benefit of Australia as a whole. The priorities provide national, regional and global dimensions which will enrich the curriculum through development of considered and focused content that fits naturally within learning areas. Incorporation of the priorities will encourage conversations between students, teachers and the wider community.

The cross-curriculum priorities include:

- Aboriginal and Torres Strait Islander Histories and Cultures
- Asia and Australia's Engagement with Asia
- Sustainability

Appendix 5 – Glossary

- A central glossary will be added to the final draft of the course for consultation.