YEARS 11&12

MATHEMATICS METHODS 4 TEACHING & LEARNING SUPPLEMENT

INDEPENDENT SCHOOLS TASMANIA

TASMANIAN CATHOLIC education office

Tasmanian Government

Teaching and Learning Supplement

MATHEMATICS METHODS 4 (MTM415117)

ADVICE FOR TEACHERS

This document helps to describe the nature and sequence of teaching and learning necessary for learners to demonstrate achievement of course outcomes.

It suggests appropriate learning activities to enable learners to develop the knowledge and skills identified in the course outcome statements.

Tasks should provide a variety and the mix of tasks should reflect the fact that different types of tasks suit different knowledge and skills, and different learning styles.

COURSE SPECIFIC ADVICE

This Teaching and Learning Supplement must be read in conjunction with the Mathematics Methods, TASC Level 4 course document, the External Assessment Specifications, the External Assessment Information Sheet and Examination Guidelines and the current TASC calculator policy document.

It contains advice to assist teachers delivering the course and can be modified as required. This Teaching and Learning Supplement is designed to support teachers who are new to or returning to teaching this course.

Mathematics Methods, TASC Level 4, is a pure mathematics course which develops an increasingly complex and sophisticated understanding of calculus and statistics. By using functions and their derivatives and integrals, and by mathematically modelling physical processes, learners develop a deep understanding of the physical world through a sound knowledge of relationships involving rates of change. Learners use statistics to describe and analyse phenomena that involve uncertainty and variation.

This course is designed for learners whose future pathways involve further study in mathematics, economics, computer science, health, social sciences and the sciences at the tertiary level.

SEQUENCE OF CONTENT

	Unit Title	Indicative Times
Unit 1	Function study	36 hours
Unit 2	Circular (trigonometric) functions	26 hours
Unit 3	Differential calculus	32 hours
Unit 4	Integral calculus	20 hours
Unit 5	Probability and statistics	20 hours

Whilst Units 1 to 4 are to be taught in that order, Unit 5 could be taught at any stage of this course.

TEACHING AND LEARNING

Unit 2

Circular (Trigonometric) Functions

This unit reviews and extend learners' previous mathematical experiences in circular (trigonometric) functions from Mathematics Methods – Foundation, before more complex transformations of functions and application scenarios are introduced. Learners are encouraged to be conversant with the three methods of representation (algebraic, graphical, and numerical) and to interrelate these methods in a variety of real world modelling scenarios.

Key Concepts

- reviewing the trigonometrical relationships involving the unit circle
- recognising and determining values of integral multiples of the angles, $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$
- recognising and sketching the graphs of $y = \sin x$, $y = \cos x$ & $y = \tan x$ on extended domains
- examining the effect of transformations of circular functions, with and without using technology, e.g. of $y = \sin x$ to $y = a\sin(bx + c) + d$
- using inverse trigonometric functions to solve trigonometric equations, e.g. $\sin(ax + b) = c$ and $\sin(ax + b) = k\cos(ax + b)$)
- investigating and solving practical applications that can be modelled using circular functions.

Examples of Learning activities

Learners:

use graphics calculators and dynamic software to investigate the shapes of circular functions, $y = \sin x$, to $y = \cos x$ & $y = \tan x$ on extended domains

sketch and interpret, with and without technology, transformed trigonometric functions of the form: e.g. $y = a\sin(bx + c) + d$, investigating the effect of changing a, b, c and *d*

explore how to develop 'general solutions' for trigonometric functions

investigate the repetitive nature of daily temperature and human pulse

identify and investigate contexts suitable for modelling by circular functions and use them in investigate practical applications. Examples could include, tidal variation, sunset and sunrise times for cities and towns in Australia, daily state/city energy usage variations

use dynamic software to investigate the superposition of two almost equal frequencies to produce 'beats', identifying the properties of the resulting function.

Unit 3

Differential calculus

(C1, 2, 3, 6)

This unit builds upon learners' existing skills sets in differential calculus from Mathematics Methods-Foundation to differentiate the functions studied in Units 1 and 2. As part of this unit, learners will study and analyse some modelled real world scenarios.

Key concepts:

- reviewing the differences between the average rate of change and an instantaneous rate of change
- reviewing the concept of a limit and the evaluation of a limit
- applying the first principles approach to determine the gradient functions of simple quadratic and cubic functions
- finding the equations of the tangent and normal to a curve at a point
- determining the derivatives of $y = x^n$ for $n \in Q, e^x, \log_e x$, $\sin x$, $\cos x$, $\tan x$, and linear combinations of these
- using the product, quotient and chain differentiation rules
- determining the differentiability of functions
- deducing the graph of the derivative function, including its domain, from the graph of a function
- applying differentiation skills to sketching graphs of functions, identifying and determining key features, such as stationary points and rates of increase or decrease
- choosing from a range of methods to identify, determine and justify the stationary points over the interval of a function,
- using differentiation skills to solve optimisation problems
- studying the applications of differentiation to model and solve real life problems
- constructing and interpreting position-time graphs, with velocity as the slope of the tangent.

Examples of Learning activities

Learners:

use motion detecting software to determine instantaneous speeds and acceleration from displacement time graphs

qualitatively interpret a graph as to the rates of increase or decrease of a function; relating these observations to the behaviour of the derivative and vice versa

determine the equation of a tangent and a normal at a point on graph

develop the derivative of $y = \sin x$, from the graph of $y = \cos x$ and vice versa by the examination of the gradient of the tangent

use technology to graphically analyse functions, their derivatives, the change of sign of the derivative and the second derivatives to explore the rules of stationary points

use functions and differential calculus to model and solve real world optimisation problems, such as maximum areas and minimum volume cases

graph the derivative of modelled real world growth and decay functions and interpret their results

determine the instantaneous rate of change of a variable with respect to another variable in real world situation, such as, rate of population change

with respect to time, change in resistance in a wire with respect to temperature, change in surface area with respect to volume, change in cost or revenue functions with respect to number of items produced

use acceleration functions to describe the changes in the velocity and, hence, the change in position of an accelerating object

use a graphics calculator or dynamic software to verify the chain rule by establishing numerical results

establish the quotient rule by applying product rule to $f(x) = h(x).(g(x))^{-1}$

apply the chain rule to real world situations involving three variables

Unit 4

Integral calculus

This unit extends learners beyond the study of differential calculus to that of integral calculus, the part of calculus which that will see learners working with integration and its application in the solution of differential equations. Using CAS graphing technology to assist, the emphasis in this unit is on applications of integration, rather than developing a large repertoire of techniques. Such applications include the determination of areas and volumes and the analysis of motion cases.

Key concepts:

- defining the indefinite integral as $\int f(x) = F(x) + c$
- determining the indefinite integrals of polynomial functions, functions of the form $f(ax + b)$ where f is x^n, for $e^x, \sin x, \cos x$ and combinations of these
- defining the definite integral as, $\int_a^b f(x)dx = F(b) - F(a)$ and its relation to the area under a curve, including an informal treatment of the fundamental theorem of calculus
- calculating the area of a region under a curve and between curves
- determining the equation of a function given its gradient or the equation of a tangent at a point on the curve
- investigating applications of integration, including areas and volumes and displacement as the integral of velocity.

Examples of Learning activities

Learners:

use technology to graphically analyse functions, their derivatives and integrals

use integration to calculate the areas under the curve of a function and between two functions, with and without technology

use motion detecting software to investigate the motion of a free falling body in terms of displacement and velocity

from a velocity time graph (or function), determine a displacement time graph (or function); interpret the result

from an acceleration time graph (or function) determine a velocity time graph (or function); interpret the result

use polynomial modelling to design and produce or laser print a two dimensional shape; use integration to determine the area of the shape

investigate the method for determining the volume of timber in a tree truck, by using several functions to describe the shape of the trunk

use integral calculus to determine the function, given the rate of growth or decay of that function; cases could include population growth, drug concentration uptake in body, etc.

Unit 5

Probability and statistics

In this unit, learners study scenarios involving discrete and continuous random variables, their representation using tables, probability functions specified by rule and defining parameters; the calculation and interpretation of central measures and measure and spread; and statistical inference for sample proportions. This unit provides learners with opportunities and techniques to examine argument and conjecture from a statistical point of view.

Key concepts:

- identifying variables, and type of variable and data (continuous and discrete)
- investigating discrete random variables, their probability distribution and calculating, interpreting and using the expected value and variance with and without technology
- using discrete random variables and associated probabilities, solve practical problems
- identifying and using the binomial distribution and associated probabilities to model data and solve practical problems
- studying the normal distribution, calculating probabilities and quantiles associated with a given normal distribution using technology
- studying statistical inference, using the concept of the sample proportion and the approximate normality of this for large samples to determine the standard deviation and hence approximate confidence intervals for the population proportion
- performing simulation experiments and collating real world data to determine various confidence intervals

Examples of Learning activities

Learners:

identify discrete variables and estimate the probabilities of their value from real data ;for example, the number of girls in families of two or three children, cars arriving at traffic lights before it turns red, etc.

examine a number of situations where the binomial distribution is appropriate and use binomial probabilities to calculate the expected value and relate it to the sample; for example, with n = 1 or 2, simulated selection of a defective object or not, etc.

classify scenarios as ones that can or cannot be modelled using a binomial or a normal distribution

consider and investigate a number of real life situations where a normal distribution can be assumed and standardise variables; for example, heights of people, masses of products, etc.

discuss different sampling techniques and possible difficulties and sources of bias; for example: 'push polling' and poor questionnaire design, as opposed to random sampling techniques

examine the variability of data collected by use of a calculator or a computer package to simulate a number of samples of data from the same distribution and look for variation in summary statistics that are estimating population proportions

conduct physical and simulated sampling experiments to explore the effects of sample size on the variation of the margin of error in confidence intervals

SUPPORTING STUDENT RESPONSES AND ELABORATIONS

Teaching and learning strategies

The inquiry model is a circular and dynamic interrelationship between planning, teaching and learning, and assessment. It is important that during the investigation learners should be able to demonstrate a degree of independence in the inquiry process.

It is important that learners receive an introduction to an issue, topic or concept to strengthen their background knowledge and understanding which will be further developed through the intensive exploration of the issue. Learners need to be able to demonstrate capacity for economic interpretation, analysis, evaluation and synthesis.

There should be a strong emphasis on assessment for learning where both the teacher and learner use understandings about progress to inform the development of the inquiry. This takes place in an on-going, continuous and cumulative manner during learning, not simply at the end.

Assessment as learning, where the inquiry itself contributes to growth in learner understanding, through reflecting upon and monitoring their own progress, allows them to inform their future learning goals and to gain an awareness of their individual learning styles.

There is scope in all course units for teachers to select learning activities in all course units for teachers to select learning activities which will engage their learners and which will challenge them appropriately. All suggested learning strategies in this course supplement can be adapted to allow learners to develop the required knowledge and skills. Some teaching and learning strategies that are particularly relevant and effective in Mathematics Methods include some of the following techniques and strategies.

Review prior learning

- brainstorming, individual, pair and group work
- learner reflection of relevant concepts and skills from F to 10 and from other units
- diagnostic tests
- formative assessment

Introduce new material

- link topic to prior mathematical knowledge, practical applications exposure to quality visual imagery/materials through a variety of media
- investigation using a range of technologies

- motivate study through the topic and the relevance to future life experiences

Provide demonstration, guided practice and application (investigations)

- teacher demonstration, modelling and peer tutoring
- 'flipping the learning', with learners reviewing and previewing class work outside of class time (videos and emulators)- this could involve the use of the DoE Fronter room (http://casas.tas.edu.au)
- teacher scaffolding to facilitate analysis of concepts
- investigation simulated real life and work scenarios
- use of online materials
- opportunities to develop modelling or problem solving skills in practical contexts

Promote independent practice and application

- research strategies and time management
- problem solving strategies
- mentoring and peer tutoring
- practice and reinforcement of learning by way of revision, worksheets, tests and demonstrations
- encourages responsibility for their own learning
- regular and meaningful feedback
- discussions, debates and learner presentations
- longer-term activities such as investigative, research and project tasks
- development of learner prepared summaries to be used in supervised assessment tasks.

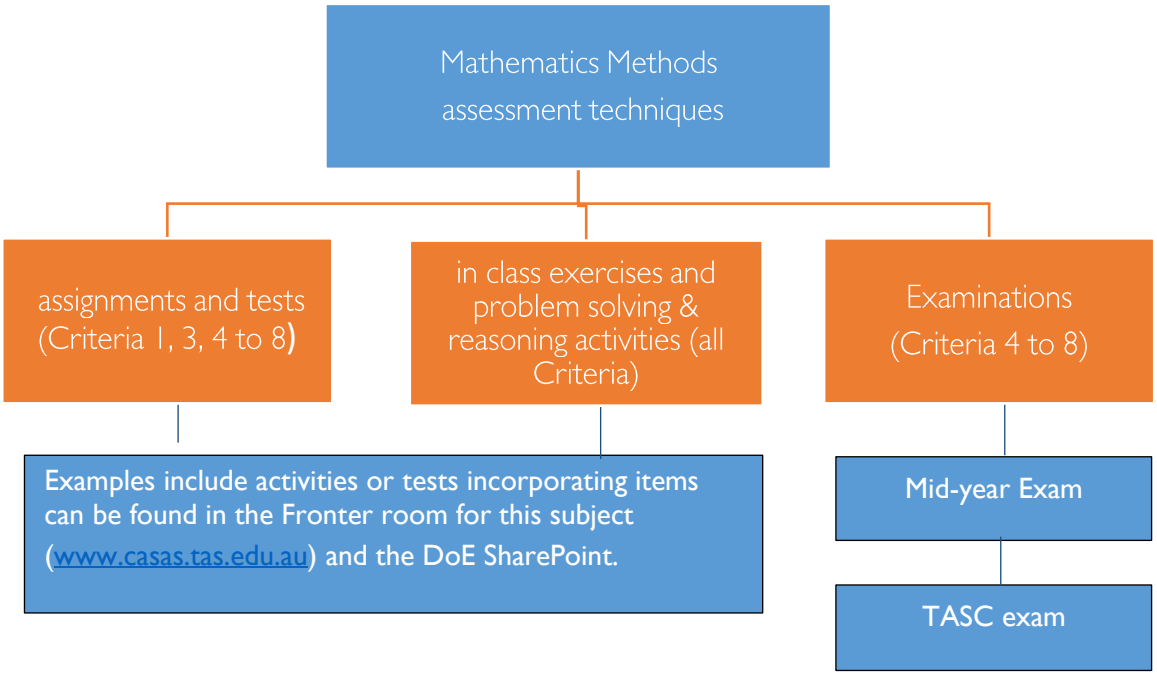
Review and rehearse

- use of assignments, structured revision times for feedback and formative assessment
- tests to help build confidence and mastery of concepts and skills

Relationship between the Course and Assessment

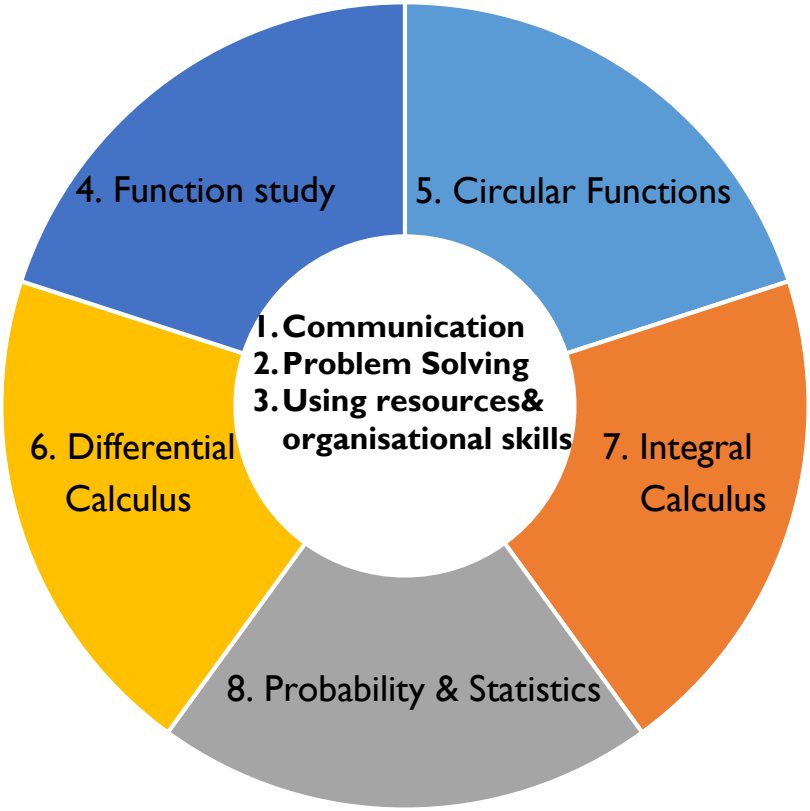
Assessment of Mathematics Methods, TASC Level 4, has both school based (internal) and externally assessed requirements, where Criteria 1 to 3 are internally assessed whilst Criteria 4 to 8 are both internally and externally assessed. Whilst Criteria 1 to 3 will be assessed against the standards document only, it is recognised that, as in most other TASC STEM subjects, that the content Criteria 4 to 8 will be assessed against a marking scale as well as against the standards.

Types of assessment



Criteria 1 to 3

Whilst the content criteria 4 to 8 are assessed within the topics and in assignments, class exercises, tests and exams, criteria 1 to 3 are internal criteria that are assessed within the context of 'in class' exercises, investigations and in assignments, with ratings given against the standards document. As they incorporate the Australian Curriculum Mathematics Methods Units 1 to 4 subject achievement standards, they are best thought of as 'content enablers'. It is expected that most learners will improve their ratings in these as the year progresses.



RESOURCES

Online support materials are provided for Mathematics Methods, including support materials found at the Curriculum Services SharePoint site

https://www.education.tas.gov.au/intranet/EYS/Curriculum-Services/Curriculum-Support-Centre/CTLResources/Pages/Mathematics.aspx

Fronter (VLE) Room

This can be accessed via www.casas.tas.edu.au

Recommended Texts and Supporting Materials

Learner Chronicles and PowerPoints

Unitised chronicles and PowerPoints are available from the Fronter room and the DoE Mathematics CTL SharePoint.

https://www.education.tas.gov.au/intranet/EYS/Curriculum-Services/Curriculum-Support-Centre/CTLResources/Pages/Mathematics.aspx

VCE/Australian Curriculum Texts

Evans, M, et al, *Cambridge Senior Mathematics Methods Units 3 and 4– Australian Curriculum/ VCE,* Cambridge 2016

Evans, M, et al, *Cambridge Senior Mathematics Methods Units 3 and 4– Australian Curriculum,* Cambridge 2016

Macbeth, Dunn et al, *Nelson Senior Mathematics Methods for the Australian Curriculum Units 3/4*, Nelson 2014

Macbeth, Dunn et al, *Nelson Senior Mathematics Methods for the Australian Curriculum Units 3/4*, Nelson 2014

Swale et al, *Maths Quest 12, Mathematical Methods*, Jacaranda, 2015

Teacher References

Strasser, D, et al, *Nelson VCE Mathematics Methods Unit 3*, Nelson Cengage 2016

Strasser, D, et al, *Nelson VCE Mathematics Methods Unit 4*, Nelson Cengage 2016

Sadler, A.J, *Mathematics Methods Unit 3*, Nelson Cengage 2015

Sadler, A.J, *Mathematics Methods Unit 4*, Nelson Cengage 2015